stiffness (**duc_kduc**). This softening modulus is used whenever the contact displacement increment is extensile. For "unloading" increments in normal displacement (i.e., the two balls approach one another), the force-displacement path is a straight line passing through the origin and the current tensile strength. When the tensile strength reduces to zero, the contact becomes unbonded and no further tension can be taken. Further, slip is allowed when the normal force becomes compressive. The magnitude of the shear force is not part of the yield criterion.

2.2.2 Smooth-Joint Model

The smooth-joint model simulates the behavior of an interface regardless of the local particle contact orientations along the interface. The behavior of a frictional or bonded joint can be modeled by assigning smooth-joint models to all contacts between particles that lie on opposite sides of the joint. This model is available to all users. The model is implemented as a user-defined contact model with C++ source files of "SmoothJoint.cpp" and "SmoothJoint.h," which are compiled and linked directly into the *PFC^{3D}* executable. The model name is **udm_SmoothJoint**, and the property names are prefixed by **sj_**.

The smooth-joint model has the following properties:

sj_dip and **sj_dd**

dip angle (θ_p) and dip direction (θ_d) [degrees]

sj_kn and **sj_ks**

normal ($\bar{k}_n$) and shear ($\bar{k}_s$) stiffness per unit area [stress/displacement]

sj_rmul radius multiplier ($\bar{\lambda}$)

sj_fric friction coefficient (μ)

sj_da dilation angle (ψ) [degrees]

sj_bmode bond mode $\left(M = \begin{cases} 0, & \text{not bonded \& never failed} \\ 1, & \text{not bonded \& failed in tension} \\ 2, & \text{not bonded \& failed in shear} \\ 3, & \text{bonded} \end{cases} \right)$

sj_bns bond normal (tensile) strength (σ_c, $\sigma_c \geq 0$) [stress]

sj_bcoh bonded system cohesion (c_b, $c_b \geq 0$) [stress]

sj_bfa bonded system friction angle (ϕ_b, $\phi_b \geq 0$) [degrees]

sj_large large-strain flag (B_l, $B_l \neq 0$ is true)

The following internal state variables of the smooth-joint model can also be accessed:

sj_active active flag (B_a, $B_a \neq 0$ is true) (read-only)

sj_xun, sj_yun, sj_zun

unit-normal vector ($\hat{\mathbf{n}}_\mathbf{j}$) defining the joint plane (read-only)

sj_A cross-sectional area (A) (read-only)

sj_rad radius ($\bar{R}$) (read-only)

sj_bss bond shear strength (τ_c, $\tau_c \geq 0$) [stress] (read-only)

sj_gap gap (g, $g > 0$ is open) (read-write)

sj_Un normal displacement (U_n) (read-only)

sj_xUs, sj_yUs, sj_zUs

shear displacement ($\mathbf{U_s}$) (read-only)

sj_Fn normal force (F_n) (read-write)

sj_xFs, sj_yFs, sj_zFs

shear force ($\mathbf{F_s}$) (read-write)

2.2.2.1 Formulation

A typical smooth-joint contact is shown in Figure 2.4. A smooth joint can only exist at a ball-ball contact. The joint geometry consists of two planar surfaces (denoted as surface 1 and surface 2) that are always parallel with each other. The plane orientation is defined by the unit-normal vector $\hat{\mathbf{n}}_\mathbf{j}$, and surface 2 is defined such that $\hat{\mathbf{n}}_\mathbf{j}$ points into surface 2. The unit normal is defined by the dip angle (θ_p) and dip direction (θ_d), using the same conventions as the **JSET** command:

$$\hat{\mathbf{n}}_\mathbf{j} = (\sin(\theta_p)\sin(\theta_d), \sin(\theta_p)\cos(\theta_d), \cos(\theta_p)) \qquad (2.22)$$

Joint orientation is fixed with respect to the global axes, and can only be changed by modifying the dip angle or dip direction. If joint orientation is changed, then the joint force orientation is changed in the same way (i.e., the joint force rotates with the joint).

Whenever the joint orientation changes (or is set initially), the two contacting entities, ball1 and ball2, are associated with the appropriate joint surfaces. The contact unit-normal vector, $\hat{\mathbf{n}}_\mathbf{c}$, is directed from the center of ball1 to the center of ball2. The dot product of $\hat{\mathbf{n}}_\mathbf{c}$ and $\hat{\mathbf{n}}_\mathbf{j}$ is used to determine in which surface each ball lies, such that ball2 lies in surface 2 if and only if $\hat{\mathbf{n}}_\mathbf{c} \cdot \hat{\mathbf{n}}_\mathbf{j} \geq 0$.

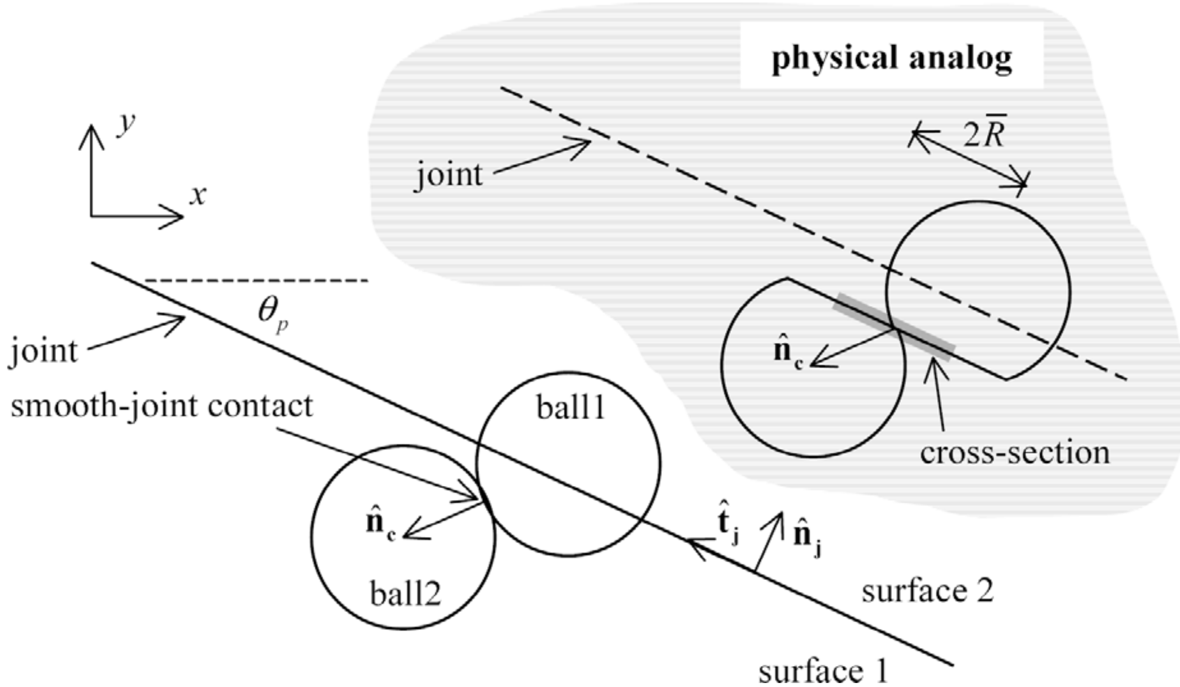

Figure 2.4 Notation used to define joint and smooth-joint contact

The following operations occur when the smooth-joint model is created:

1. The contact model and parallel bond (if present) are deleted and replaced by the smooth-joint contact model with no parallel bond.
2. All properties, except for dip angle and dip direction, are inherited from the properties of the contact and the two contacting entities via

$$\bar{\lambda} = \begin{cases} \bar{\lambda}_{pb}, & B_{pb} \neq 0 \\ 1.0, & B_{pb} = 0 \end{cases}$$

$$\bar{k}_n = (k^n/A) + \bar{k}^n$$

$$\bar{k}_s = (k^s/A) + \bar{k}^s$$

$$\mu = \mu_c$$

$$\psi = 0.0$$

$$M = \begin{cases} 3, & B_{pb} \neq 0 \text{ or } B_{cb} \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\sigma_c = \begin{cases} \min((\phi_n/A), \bar{\sigma}_c), & B_{pb} \neq 0 \text{ and } B_{cb} \neq 0 \\ \bar{\sigma}_c, & B_{pb} \neq 0 \text{ and } B_{cb} = 0 \\ (\phi_n/A), & B_{pb} = 0 \text{ and } B_{cb} \neq 0 \\ 0.0, & B_{pb} = 0 \text{ and } B_{cb} = 0 \end{cases}$$

$$c_b = \begin{cases} \min((\phi_s/A), \bar{\tau}_c), & B_{pb} \neq 0 \text{ and } B_{cb} \neq 0 \\ \bar{\tau}_c, & B_{pb} \neq 0 \text{ and } B_{cb} = 0 \\ (\phi_s/A), & B_{pb} = 0 \text{ and } B_{cb} \neq 0 \\ 0.0, & B_{pb} = 0 \text{ and } B_{cb} = 0 \end{cases}$$

$$\phi_b = 0.0$$

where B_{pb} and B_{cb} are parallel- and contact-bond flags, respectively (nonzero if and only if a bond is present). The inherited properties can be overridden by assigning smooth-joint properties directly.

3. The force, displacement and gap are set to zero.

The smooth joint can be envisioned as a set of elastic springs uniformly distributed over a circular cross-section, centered at the contact point and oriented parallel with the joint plane. The area of the smooth-joint cross section is given by

$$A = \pi \bar{R}^2 \tag{2.23}$$

where smooth-joint radius $\bar{R} = \bar{\lambda} \min(R^{(A)}, R^{(B)})$, with $R^{(A)}$ and $R^{(B)}$ being the particle radii. Let $\mathbf{U}$ be the translational displacement vector of surface 2 relative to surface 1, and let $\mathbf{F}$ be the force vector acting on surface 2. These vectors can be expressed as

$$\begin{aligned} \mathbf{U} &= U_n \hat{\mathbf{n}}_\mathbf{j} + \mathbf{U_s} \\ \mathbf{F} &= F_n \hat{\mathbf{n}}_\mathbf{j} + \mathbf{F_s} \end{aligned} \tag{2.24}$$

where $\mathbf{U_s}$ and $\mathbf{F_s}$ are the displacement vector and force vector, respectively, that lie in the joint plane. Positive U_n denotes overlap, and positive F_n denotes compression.

The contact force-displacement law provides either Coulomb sliding with dilation ($M < 3$) or bonded behavior ($M = 3$, a bond is added by setting $M = 3$). At the start of each cycle, the active status of the model is determined. If in small-strain mode ($B_l = 0$, the default), then active is true; otherwise, active is true if and only if either there is a nonzero overlap between the two contacting particles or the joint is bonded. If the model is inactive, then the parent contact may be deleted at the discretion of *PFC3D*, and the displacement and gap are set to zero. If the model is active, then the relative displacement increment between the two ball surfaces is decomposed into components normal and tangential to the joint surfaces (ΔU_n and $\Delta \mathbf{U_s}$) and the total displacement is updated. The elastic portion of the displacement increment (ΔU_n^e and $\Delta \mathbf{U_s}^e$) is determined based on the value of the gap: if bonded, then the entire displacement increment is elastic; if not bonded, then only the portion of the displacement increment that occurs while the gap is negative is elastic. The elastic components of the displacement increment are multiplied by the smooth-joint normal and shear stiffnesses to produce increments of joint force. The force is updated via

$$
\begin{aligned}
F_n &:= F_n + \bar{k}_n A \Delta U_n^e \\
\mathbf{F_s'} &:= \mathbf{F_s} - \bar{k}_s A \Delta \mathbf{U_s}^e
\end{aligned}
\tag{2.25}
$$

The joint is in one of two modes: unbonded ($M < 3$) or bonded ($M = 3$). The behavior differs for each mode as follows.

Unbonded joint (Figure 2.5):

If $|\mathbf{F_s'}| \le (F_s^* = \mu F_n)$, then $|\mathbf{F_s}| = |\mathbf{F_s'}|$. Otherwise, sliding is assumed to occur, and shear displacement during sliding produces an increase in normal force:

$$
\begin{aligned}
|\mathbf{F_s}| &= F_s^* \\
F_n &:= F_n + [\Delta U_s^* \tan\psi] \bar{k}_n A = F_n + \left(\frac{|\mathbf{F_s'}| - F_s^*}{\bar{k}_s} \right) \bar{k}_n \tan\psi
\end{aligned}
\tag{2.26}
$$

where μ is the friction coefficient, and ψ is the dilation angle.

Bonded joint (Figure 2.6):

If $F_n \le -\sigma_c A$, then the bond breaks in tension ($M = 1$ and $F_n = |\mathbf{F_s}| = 0$). Otherwise, the bond remains intact and F_n is not altered. If $|\mathbf{F_s'}| \ge \tau_c A$, then the bond breaks in shear ($M = 2$ and if the bond is in tension, then $F_n = |\mathbf{F_s}| = 0$; otherwise, F_n is not altered and $|\mathbf{F_s}| \le F_s^*$). Otherwise, the bond remains intact and $|\mathbf{F_s}| = |\mathbf{F_s'}|$. In the above expressions, σ_c and τ_c are the bond normal and shear strengths, respectively, and A is the bond cross-sectional area.

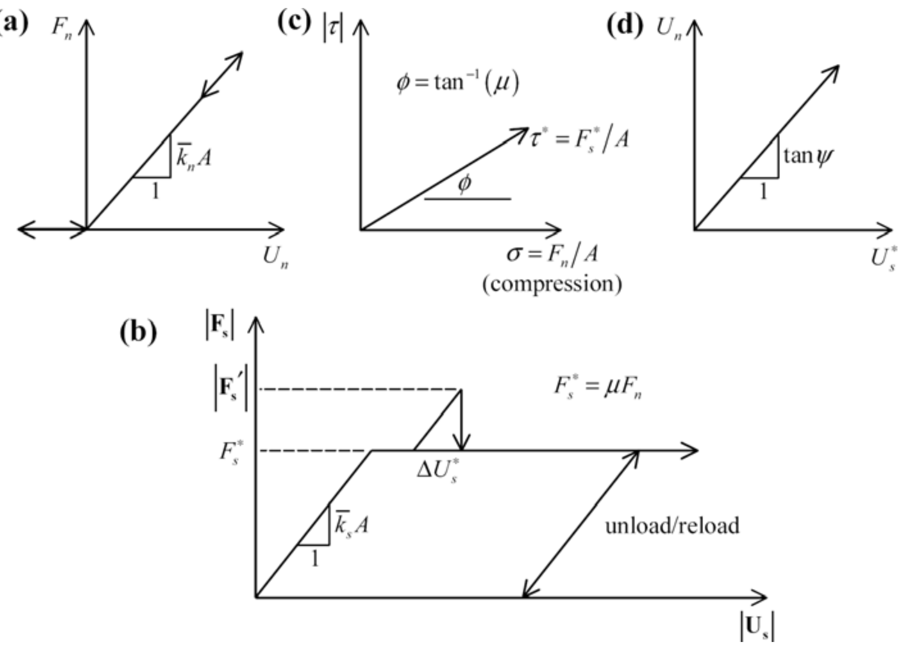

Figure 2.5 Force-displacement law for an unbonded joint: (a) normal force versus normal displacement, (b) shear force versus shear displacement, (c) strength envelope and (d) normal displacement versus shear displacement during sliding

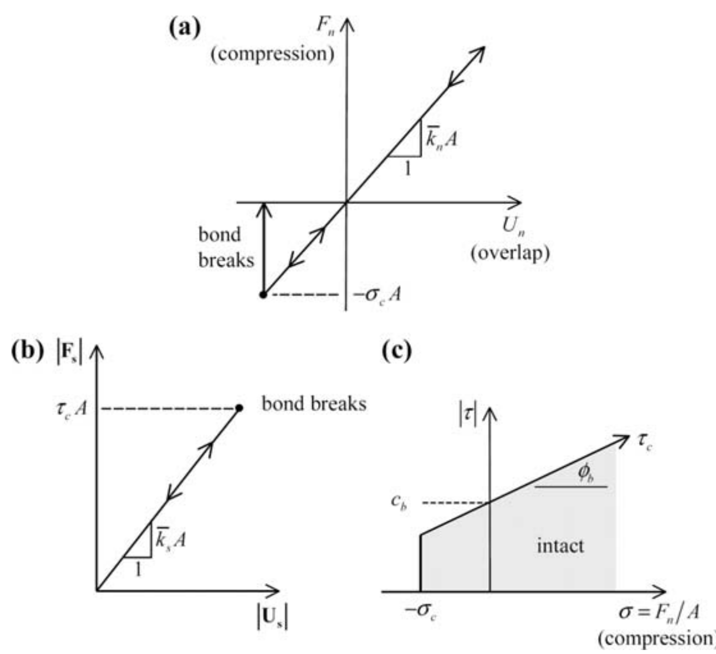

Figure 2.6 Force-displacement law for a bonded joint: (a) normal force versus normal displacement, (b) shear force versus shear displacement and (c) strength envelope

2.2.2.2 Creating a Single Smooth-Joint Contact

This example (see Example 2.1) demonstrates that the smooth-joint model operates correctly at a single contact. It shows how one can define a local planar surface with an orientation that does not coincide with the contact orientation. The model consists of two unit-radius particles oriented at 45 degrees from the $y = 0$ plane (see Figure 2.7). A smooth joint with unit normal aligned with the global y-axis is assigned to the contact. It is assigned normal and shear stiffnesses of unity, a radius multiplier of 0.56419 (to yield an area of unity), and a friction coefficient of zero. All degrees of freedom of both particles are fixed. The upper ball is first moved downward toward the $y = 0$ plane, and then moved parallel with the $y = 0$ plane. The normal and shear forces with respect to the joint plane are compressive ($F_n > 0$) and zero ($\mathbf{F_s} \approx 0$), respectively (see Figure 2.8). The friction coefficient is set to unity, and the upper ball continues to move parallel with the $y = 0$ plane. The normal force remains constant, and the shear-force magnitude increases until it equals the normal force, and then also remains constant (see Figure 2.9).

Example 2.1 Smooth joint at a single contact

```
;fname: SmoothJoint3d.dat
new
def setup
  _b2x = sqrt(2.0)
  _b2y = sqrt(2.0)
end
setup
ball id=1 rad=1.0 x=0.0  y=0.0  z=0.0
ball id=2 rad=1.0 x=_b2x y=_b2y z=0.0
prop dens=1.0
model udm_SmoothJoint
prop sj_dip=90.0 sj_dd=0.0 sj_kn=1.0 sj_ks=1.0 sj_fric=0.0 sj_da=0.0
prop sj_rmul=0.56419  ; to give A = 1.0, same as SmoothJoint2d.dat

plot create the_system
plot set title text 'Smooth-joint contact, normal aligned with y-axis.'
plot set rot 90 0 0
plot add ball yellow
plot add cforce black red
plot add contact blue red
plot add axes
plot show

fix x y z xspin yspin zspin
prop yvel=-1.0e-5 range id=2
cycle 70
prop yvel=0.0 range id=2
prop xvel=-1.0e-4 range id=2
cycle 5000
```

```
print contact prop  ; Note that sj_Fn > 0, _xFs ~ 0.0
pause
prop sj_fric=1.0
cycle 8500
print contact prop  ; Note that _xFs = 1.0 * sj_Fn
return
;EOF: SmoothJoint3d.dat
```

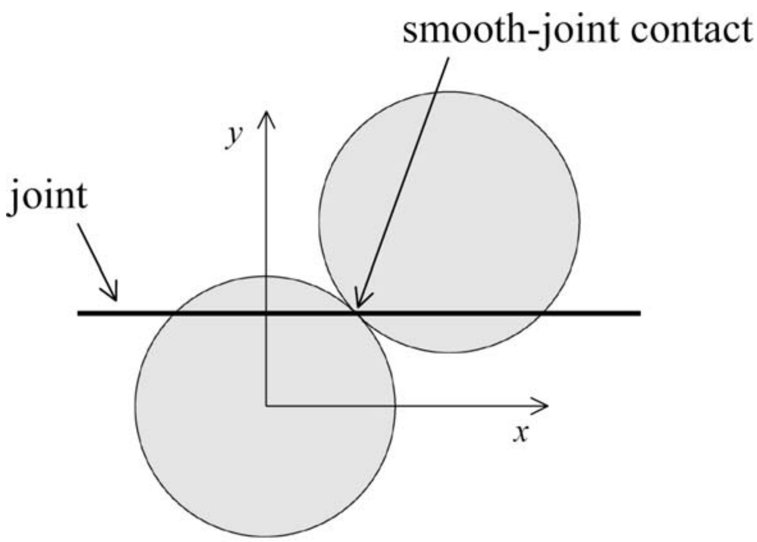

Figure 2.7 Initial system configuration

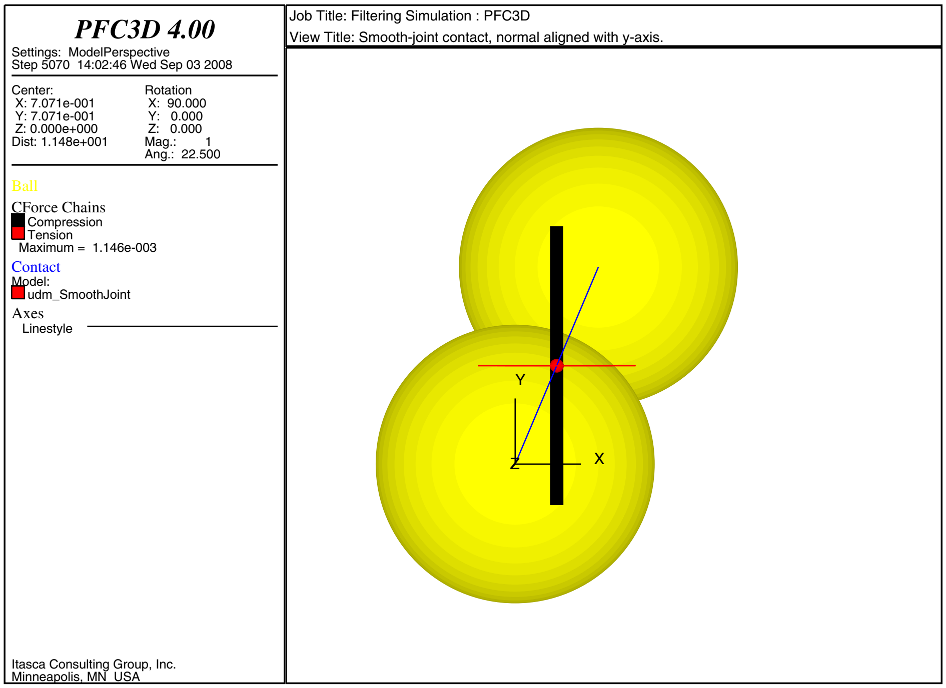

Figure 2.8 Contact force in PFC^{3D} model after downward and left-horizontal motion of upper particle (friction coefficient of zero)

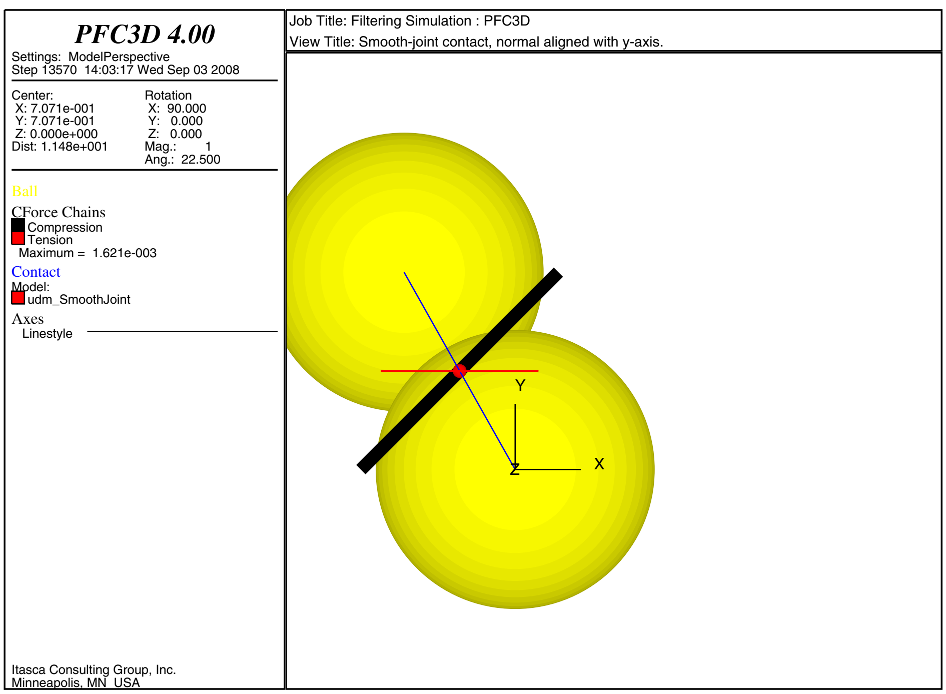

Figure 2.9 Contact force in PFC^{3D} model after further left-horizontal motion of upper particle (friction coefficient of unity)

2.2.2.3 Modeling Joints as Collections of Smooth-Joint Contacts

The *PFC Fishtank* supports modeling joints as collections of smooth-joint contacts. A bonded-particle model is the starting point for the following procedure. A collection of disk-shaped joints and their properties is defined in two ASCII files that are read into the code by **jt_Read**, such that joints are numbered sequentially starting from one. The function **sj_Make** creates a set of smooth-joint contacts by adding the joints one at a time, starting with the first joint. For each joint, contacts that satisfy the criterion of the **c_ondisk()** *FISH* intrinsic, and do not already have a smooth-joint contact model, are assigned a smooth-joint contact model with properties of the joint and a joint-set number equal to the joint number. The macroscopic joints can be visualized with the **jt_pi_geom** plot item, and the individual smooth-joint contacts can be visualized with the built-in contact plot item.

Deformation that occurs after the smooth-joint contacts have been assigned may induce formation of new contacts that satisfy the criterion of the **c_ondisk()** *FISH* intrinsic. These new contacts represent new surface interactions, and may have different properties than the initial surfaces. Therefore, there are two sets of joint properties that correspond with the initial and new surfaces. The new properties are assumed equal to the initial properties unless the function **jt_ReadNewProp** is called. If these new contacts are not assigned a smooth-joint contact model, then they will behave as interacting asperities because they will be assigned the default contact model, and spurious contact forces will develop between them. Such asperity lockup can be prevented by registering the function **sj_Add** to be called whenever a new contact is created.

The first example (see the data files in the directory “fist\templates\LdB\3d\sj-3d,” driven by “Solid1-sj.dvr”) adds a single through-going joint to a bonded-particle specimen, and then performs an unconfined compression test on the specimen to demonstrate that the joint is behaving properly. A bonded-particle model of Lac du Bonnet granite with a resolution of approximately 10 particles across the specimen width is first created by setting **mg_Rmin** = 1.22e-3 in “resolution.dat,” and then calling “Solid-spc.dvr.” The specimen is a cylinder with height and diameter of 30 mm, and the specimen axis is aligned with the global *y*-axis. The 20 mm radius joint is placed at the center of the specimen and oriented with a −60 degree dip angle and 0 degree dip direction. The joint is shown in Figure 2.10.

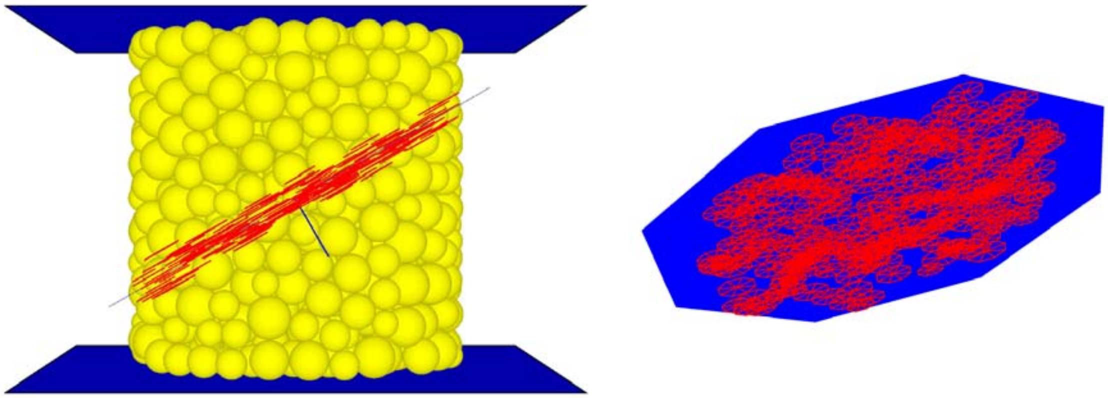

Figure 2.10 *PFC3D specimen after insertion of joint (left: side view, right: rotated view showing macroscopic joint and smooth-joint contacts drawn as 8-spoked disks)*

The joint properties are set to

$$(\bar{k_n} = \bar{k_s}) \approx E/L \approx E/\tilde{D} = \frac{70\times10^9 \mathrm{N/m^2}}{3\times10^{-3}\mathrm{m}} = 2.3 \times 10^{13}\ \mathrm{N/m^3}$$

$$\bar{\lambda} = 1.0 \text{ (set by } \mathbf{sj\_Make}\text{)}$$

$$\mu = 0.5 \tag{2.27}$$

$$\psi = 0$$

$$M = 0, \sigma_c = c_b = \phi_b = 0.0 \text{ (not bonded)}$$

where the stiffnesses per unit area are chosen to approximately equal the stiffnesses per unit area of a one-particle width slice of the bonded material. (The relation is derived by noting that stiffness equals AE/L, the macroscopic modulus of the Lac du Bonnet granite is 70 GPa, and the average particle diameter is 3 mm.)

The first step of an unconfined compression test requires that the specimen be seated by applying a small axial stress (defined by **mt_tas**). With an initial axial stress of -0.1 MPa and a joint friction coefficient of 0.5, the joint begins to slide during the seating process (see Figure 2.11). If the joint friction coefficient is increased to 1.0, then the seating is successful and a test can be performed.

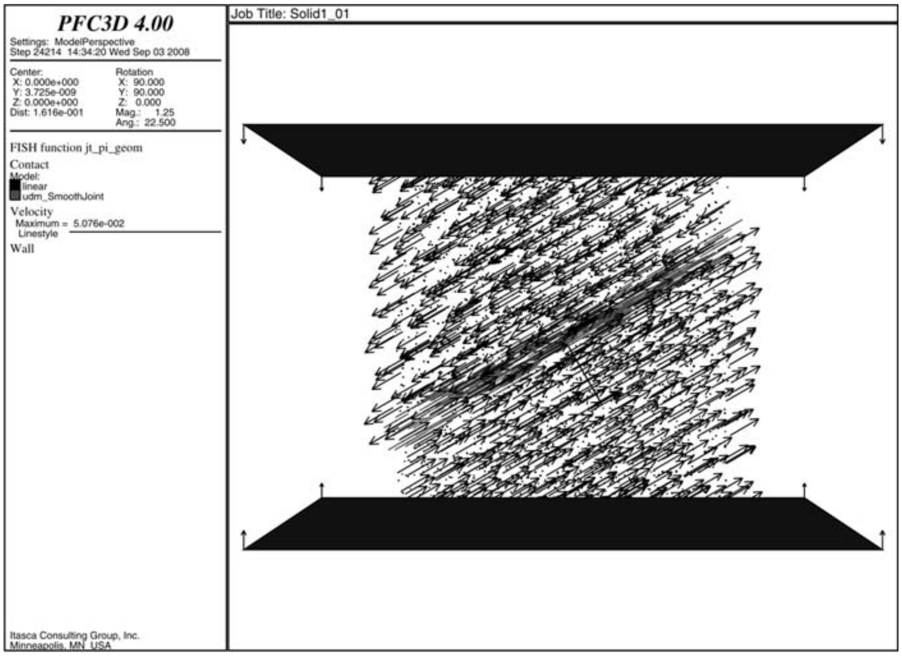

Figure 2.11 ***Velocity field while seating the*** *PFC^{3D}* ***specimen (friction coefficient of 0.5) with an axial stress of*** -0.1 ***MPa (The specimen is failing by sliding along the joint.)***

The second example (see the data files in the directory "fist\templates\LdB\3d\sj-3d," driven by "Solid2-sj.dvr") adds seven through-going joints at a uniform spacing to a bonded-particle specimen, and then performs a compression test at 1.0 MPa confinement on the specimen. The bonded-particle model of Lac du Bonnet granite (described in the first example) with a resolution of approximately 20 particles across the specimen width is first created. Seven through-going joints at a 10 mm spacing and oriented with a −60 degree dip angle and 0 degree dip direction are inserted. The joints are shown in Figure 2.12.

The joint properties are the same as those used in the first example, except that the friction coefficient is set to zero. The compression test reaches a steady-state axial stress of 3.3 MPa. At this point, a mechanism has formed, whereby upper and lower blocks are sliding (see Figure 2.13). Plots of contact forces (not shown here) confirm that no shear forces are acting on the joint planes.

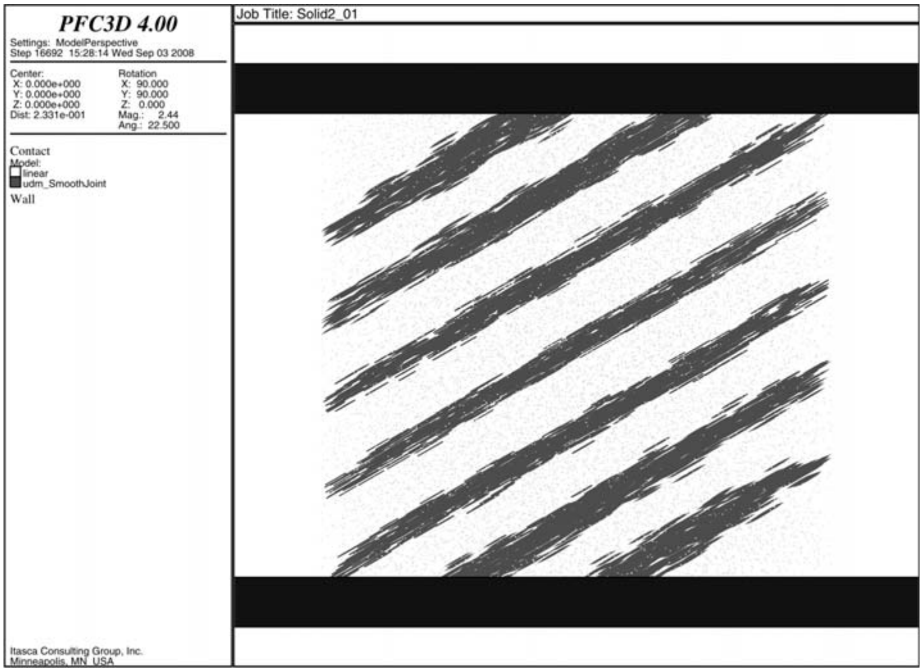

Figure 2.12** PFC^{3D}* ***specimen after insertion of the seven joints (consisting of 5479 smooth-joint contacts)

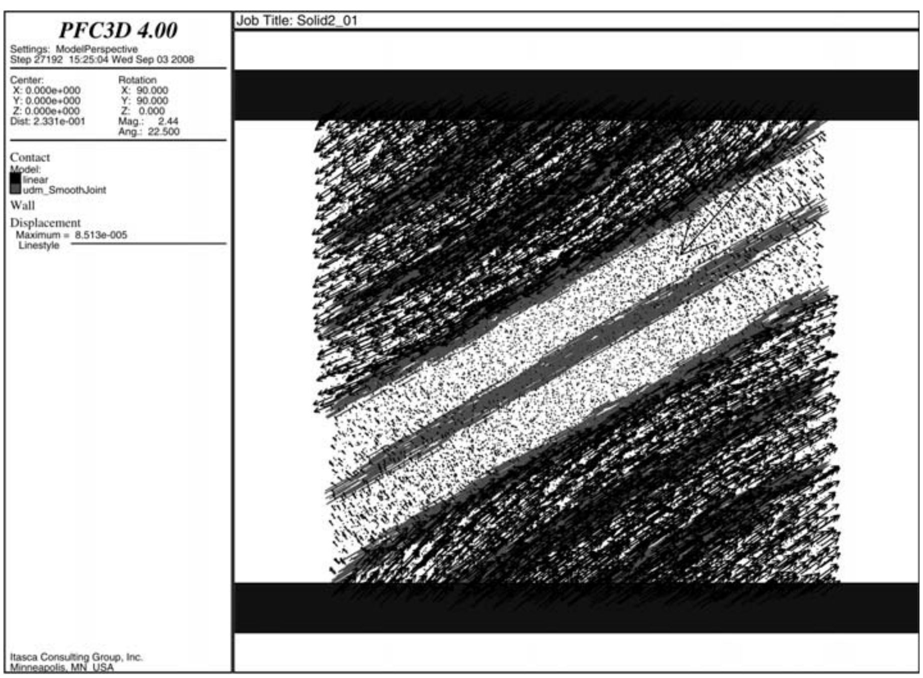

Figure 2.13** **Displacement field in the *PFC^{3D}* ***specimen at axial strain of 0.05%***

2.2.2.4 Demonstration of Smooth-Joint Contact Logic

This example (see the data files in the directory "fist\templates\LdB\3d\sj-3d\LargeSlide," driven by "LargeSlide.dvr") demonstrates large sliding motion on a joint that is represented as a collection of smooth-joint contacts. A cylindrical specimen is created, removed from the material vessel and allowed to expand. A through-going joint oriented at 30 degrees from the horizontal with a friction coefficient of one is created. The smooth joints are run in large-strain mode (activated by the command **PROP sj_large 1.0**). The joint is represented by 52 smooth-joint contacts. Initially, only 14 of these contacts are active. The total number of smooth-joint contacts, and the number of these that are active, are given by **sj_num** and **sj_numActive**, respectively. A vertical wall is added to the left of the specimen to limit the lateral motion of the top block. The bottom layer of particles is fully fixed. Gravity is activated, and the system is allowed to reach static equilibrium. The blocks are stable for a joint friction coefficient of one. The joint friction coefficient is reduced to zero, and the top block begins to slide until it butts up against the left wall (see Figure 2.14). At this point, the model contains 16 active smooth-joint contacts. The new contacts were added by the function **sj_Add** (described in Section 2.2.2.3), and the old contacts were removed automatically because the smooth joints were run in large-strain mode.

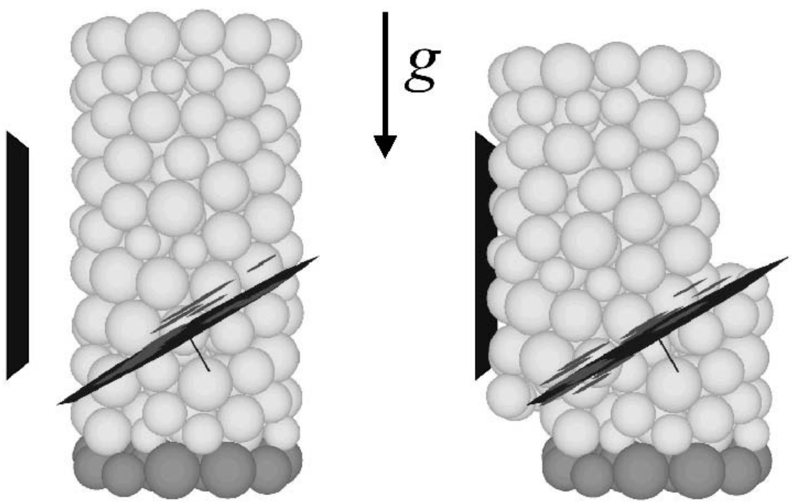

Figure 2.14 Stable *PFC^{3D}* ***models with joint friction coefficients of 1.0 (left) and 0.0 (right)***

2.2.2.5 Joint Mechanical Properties

The information in this section is taken from Itasca (2007), which should be consulted for more information.

There are well-established methods for the characterization of joint mechanical properties (Barton 1976; Stephansson 1985; Barton and Stephansson 1990). Joint properties are conventionally derived from laboratory testing (e.g., triaxial, direct shear or tilt tests). These tests can produce physical properties for joint friction angle, cohesion, dilation angle and tensile strength, as well as joint normal and shear stiffnesses. The joint cohesion and friction angle correspond to the parameters in the Coulomb strength criterion.

Values for normal and shear stiffnesses for rock joints range from 10 to 100 MPa/m for joints with soft clay in-filling, to over 100 GPa/m for tight joints in granite and basalt. Published data on stiffness properties for rock joints is limited; summaries of data can be found in Kulhawy (1975), Rosso (1976) and Bandis et al. (1983). Additional data has been generated by the various research programs for nuclear waste disposal. Most of this data can be obtained from the web sites of the nuclear waste disposal management companies.

Published strength properties are more readily available for joints than for stiffness properties. Summaries can be found, for example, in Jaeger and Cook (1969), Kulhawy (1975) and Barton (1976). Friction angles can vary from less than 10 degrees for smooth joints in weak rock such as tuff, to over 50 degrees for rough joints in hard rock such as granite. Joint cohesion can range from zero to values approaching the compressive strength of the surrounding rock.

Joint properties measured in the laboratory typically are not representative of those for real joints in the field. Scale-dependence of joint properties is a major question in rock mechanics. Often, the only way to guide the choice of appropriate parameters is by comparison to similar joint properties derived from field tests. However, field test observations are extremely limited (some results are reported by Kulhawy 1975).

Approximate stiffness values can be back-calculated from information on the deformability and joint structure in the jointed rock mass and the deformability of the intact rock. If the jointed rock mass is assumed to have the same deformational response as an equivalent elastic continuum, then relations can be derived between jointed rock properties and equivalent continuum properties (see Itasca 2007).

Several expressions have been derived for two- and three-dimensional characterizations and multiple joint sets. References for these derivations can be found in Singh (1973), Gerrard (1982(a) and (b)) and Fossum (1985). Analytical solutions for estimating the equivalent properties of fractured rock masses are limited to simple fracture geometries (Salamon 1968; Singh 1973; Amadei and Goodman 1981; Amadei 1988).

There is a limit to the maximum joint stiffnesses ($\bar{k}_n$ and $\bar{k}_s$) that are reasonable to use in a *PFC*^{3D} model. If the physical normal and shear stiffnesses are less than ten times the equivalent stiffnesses of adjacent contacts, then there is no problem in using physical values. (The equivalent stiffness is the largest total stiffness per unit area of the contacts adjacent to the joint – e.g., the total stiffness per unit area of a parallel-bonded contact is equal to the parallel-bond stiffness per unit area plus the contact stiffness divided by the contact area.) If the ratio is more than ten, the solution time will be significantly longer than for the case in which the ratio is limited to ten, without much change in the behavior of the system. To improve solution efficiency, serious consideration should be given to reducing supplied values of joint normal and shear stiffnesses. There may also be problems with particle interpenetration if the joint normal stiffness is very low. A rough estimate should be made of the joint normal displacement that would result from the application of typical stresses in the system ($u = \sigma/\bar{k}_n$). This displacement should be small compared to typical particle size. If it is greater than, say, 10% of an adjacent particle diameter, then either there is an error in one of the numbers or the stiffness should be increased.

2.2.3 Displacement-Softening Model

The displacement-softening model provides bonded behavior in which the strength is reduced as a function of applied displacement. This model is available to all users. The model is implemented as a user-defined contact model with C++ source files of "softening.cpp" and "softening.h," which are compiled and linked directly into the *PFC3D* executable. The model name is **udm_softening**, and the property names are prefixed by **sof_**.

The displacement-softening model has the following properties:

sof_broken	$= 1.0$ if the strengths, F_c^k, $(k = n, s)$ have reduced to zero.
sof_fric	friction coefficient
sof_fsmax	shear strength, F_c^s (a positive number)
sof_ftmax	tensile strength, F_c^n (a positive number)
sof_knc	normal stiffness in compression
sof_knt	normal stiffness in tension
sof_ks	shear stiffness
sof_rfric	residual friction coefficient
sof_uplim	accumulated plastic displacement for which the bond strength softens to zero (see Figure 2.15), $U_{p\max}$

In addition, the following calculated properties record the state of the model, but cannot be set by the user:

sof_softened	$= U_p/U_{p\max}$; **sof_softened** is equal to 0.0 if contact has never yielded.
sof_uplas	currently accumulated plastic displacement (see Figure 2.15), U_p; and