



ITASCA

# Effects of stress and induced cracking on the static and dynamic moduli of rock

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# Acknowledgements

- Work was performed as part of the **SAFETI** (Seismic validation of 3-D thermo-mechanical models for the prediction of rock damage around radioactive waste packages in geological repositories) project, full reference given in paper.
- Work involved both experimental & numerical investigations
  - Experimental work performed by Baker & Pettitt
  - Numerical work performed by Potyondy & Hazzard

# Objective

- We want to quantify damage induced by stress application, which produces cracks that decrease modulus & increase permeability.
- Methods to quantify this damage include:
  - Recording microseismicity, measuring wave velocities, or measuring changing moduli directly *to infer damage*
  - Counting cracks on thin sections *to see damage*
- We propose that a bonded-particle model (BPM) can mimic stress-induced damage formation, study this by
  - Simulating a laboratory experiment and comparing numerical and laboratory results

# What was done

- Perform laboratory experiment on cubic sandstone specimen
  - Apply three-phase loading path, induce some damage
  - Measure evolution of elastic stiffnesses
    - Static stiffnesses from stress probes
    - Dynamic stiffnesses from wave-velocity probes
  - Quantify the stiffness anisotropy as a function of both applied stress and induced damage
- Simulate this experiment with BPM
- Compare evolution of elastic stiffnesses

# Stiffness anisotropy in BPM

- Elastic stiffnesses are affected by both **stress & damage**.
  - Increasing the mean stress, increases the coordination number, which increases the stiffness magnitude but does not induce anisotropy.
  - Anisotropy arises from deviatoric stress, which modifies the force-chain fabric:
    - increasing contacts in compression direction, and
    - decreasing contacts orthogonal to compression direction.
  - Damage, in form of bond breakages, enhances these effects.
- Similar effects occur in rock, dependent on microstructure

# Characterizing elastic anisotropy

- Elastic constants relate stress to strain

$$\sigma_{ij} = c_{ijkl} \varepsilon_{kl} \quad (c_{ijkl} \text{ are elastic stiffnesses})$$

$$\varepsilon_{ij} = s_{ijkl} \sigma_{kl} \quad (s_{ijkl} \text{ are elastic compliances})$$

tensor notation

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$$\sigma_q = c_{qr} e_r \quad (c_{qr} \text{ are elastic stiffnesses})$$

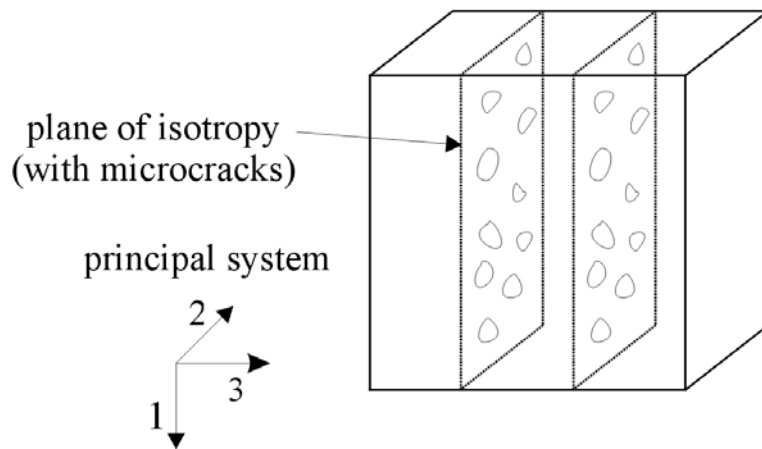
$$e_q = s_{qr} \sigma_r \quad (s_{qr} \text{ are elastic compliances})$$

Voigt notation

- Linear-elastic theory
  - Stress-strain relations are linear: 36 independent components
  - Deformations are reversible: 21 independent components
  - Symmetry considerations reduce the number much further...

# Characterizing elastic anisotropy

- Isotropic material: 2 independent components
- **Transversely isotropic material: 5 independent components**

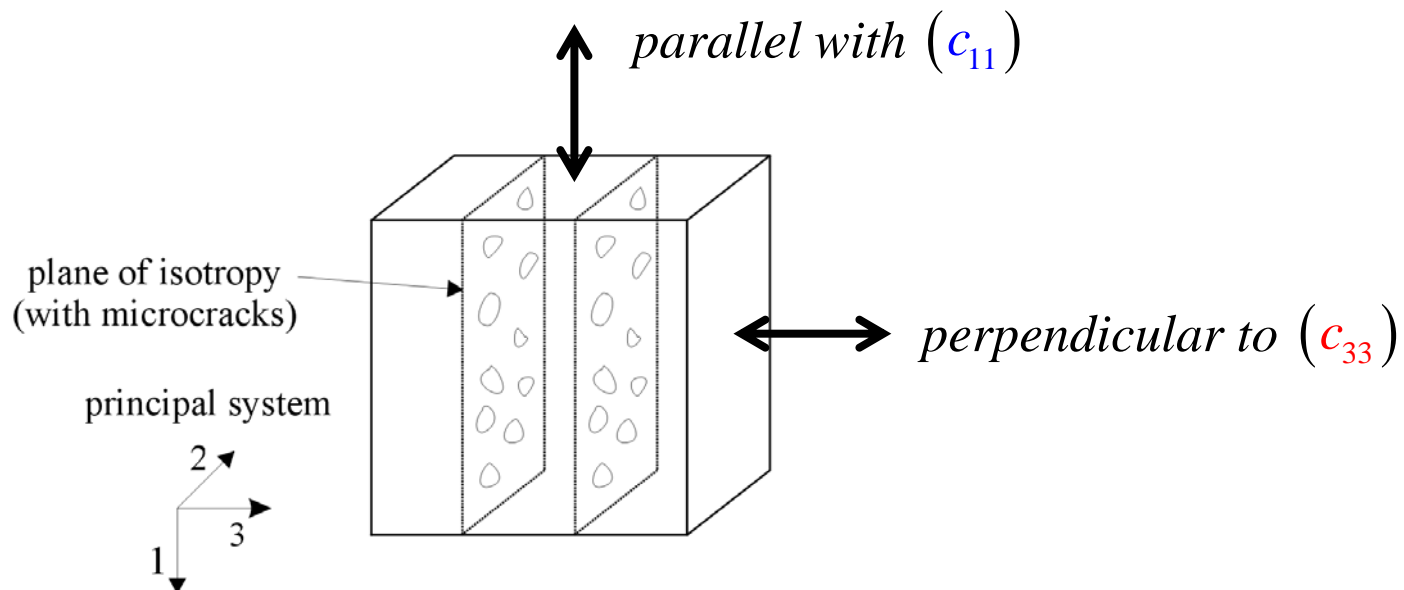


$$c_{qr} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & & & \\ c_{12} & c_{11} & c_{13} & & & \\ c_{13} & c_{13} & c_{33} & & & \\ & & & c_{44} & 0 & 0 \\ & [0]_{3 \times 3} & & 0 & c_{44} & 0 \\ & & & 0 & 0 & c_{66} \end{bmatrix}$$

$$\text{with } c_{66} = \frac{1}{2}(c_{11} - c_{12})$$

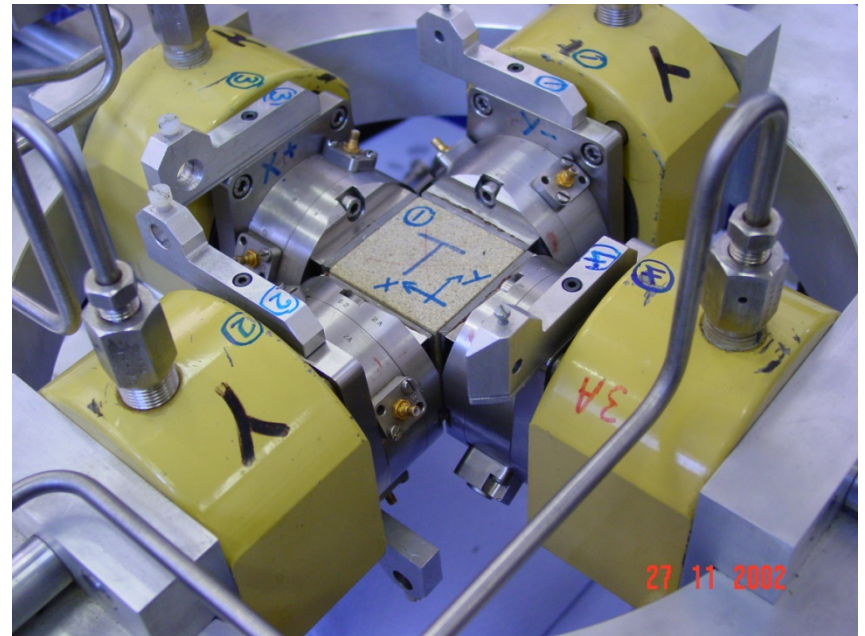
# Transversely isotropic material

- Discussion will focus on  $c_{11}$  and  $c_{33}$
- Measure of stiffness:
  - *parallel with* & *perpendicular to* planes of isotropy



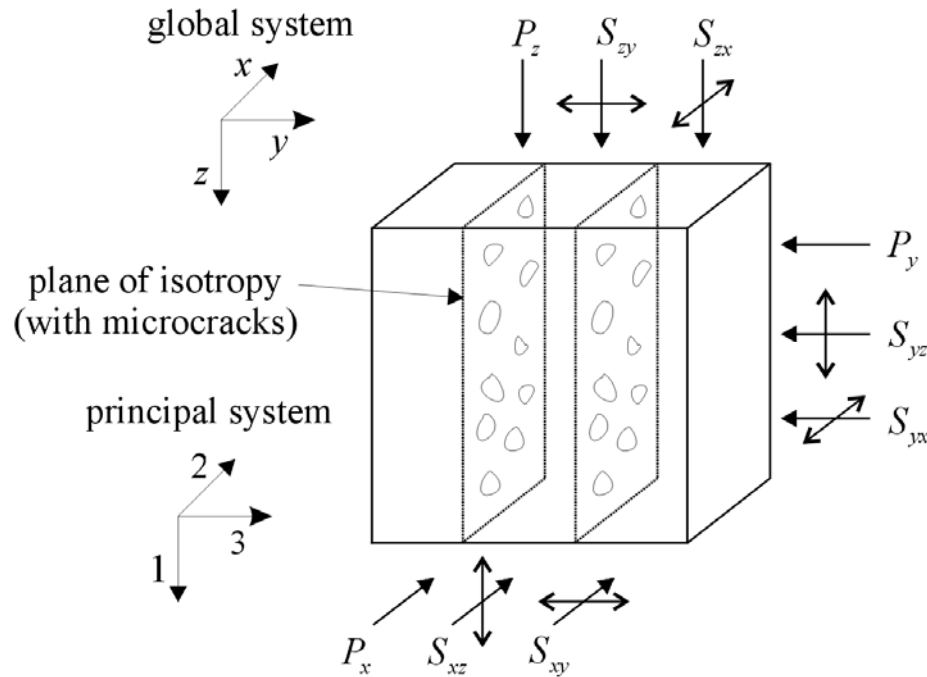
# Laboratory Experiment

- Polyaxial (true triaxial) loading cell
- 51-mm cube of Crosland Hill sandstone
- Measure stress/strain & ultrasonic velocities



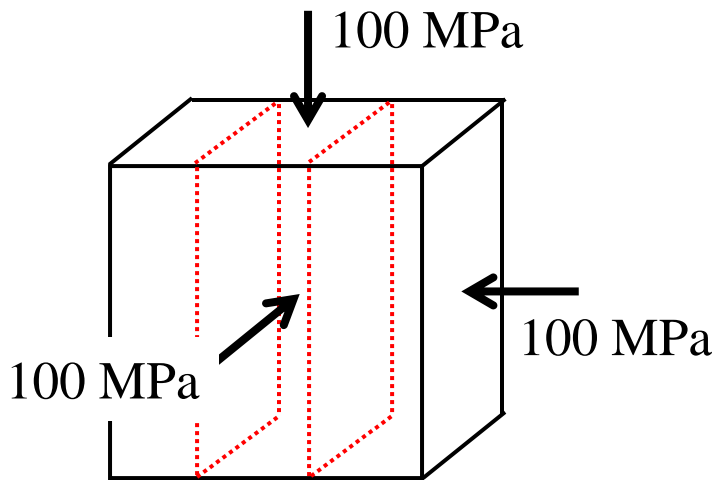
# Laboratory Experiment

- Measure ultrasonic velocities, each axis:
  - one P-wave, two polarized S-wave components

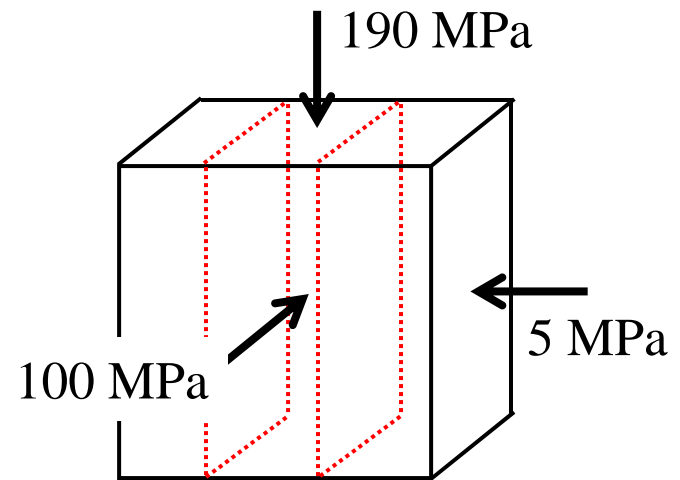


# Laboratory Experiment

- Loading history (induce cracking in 1-2 plane)
  - Hydrostatic and fracturing phases



hydrostatic phase



fracturing phase

# PFC3D Experiment

- Bonded-particle model
  - Spherical grains joined by parallel bonds
  - Tested in sleeved-triaxial cell
  - Matches elastic constants & compressive strength of Crosland Hill sandstone

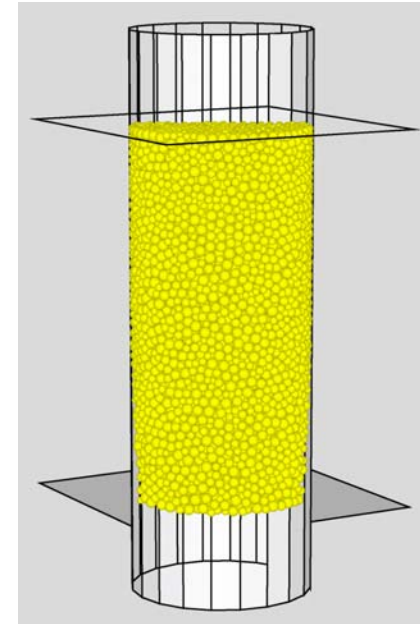


Table 1. *PFC*<sup>3D</sup> model microproperties\* for Crosland Hill sandstone.

Grains	Cement
$\rho = 3798 \text{ kg/m}^3$	
$(D_{\max} / D_{\min}) = 1.66$	$\bar{\lambda} = 1.0$
$E_c = 20.1 \text{ GPa}$	$\bar{E}_c = 20.1 \text{ GPa}$
$(k_n / k_s) = 2.1$	$(\bar{k}^n / \bar{k}^s) = 2.1$
$\mu = 0.5$	$\bar{\sigma}_c = \bar{\tau}_c = 112 \pm 42 \text{ MPa (mean } \pm \text{ std. dev.)}$

\* Symbols defined in Potyondy & Cundall (2004)

# PFC3D Experiment

- Bonded-particle model
  - Spherical grains joined by parallel bonds
  - Tested in sleeved-triaxial cell
  - Matches elastic constants & compressive strength of Crosland Hill sandstone

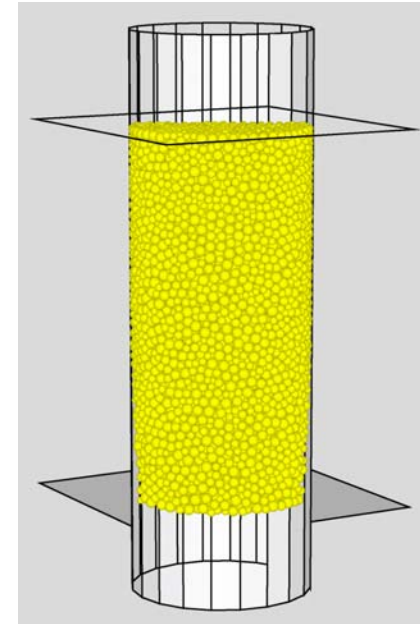
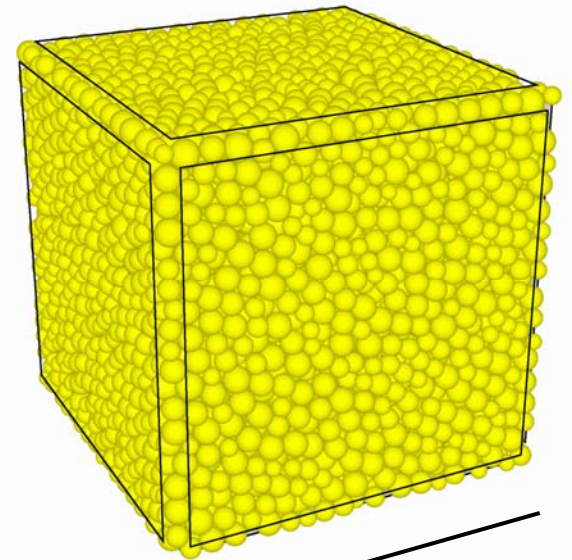


Table 2. Macro-properties measured at 1-MPa confinement.

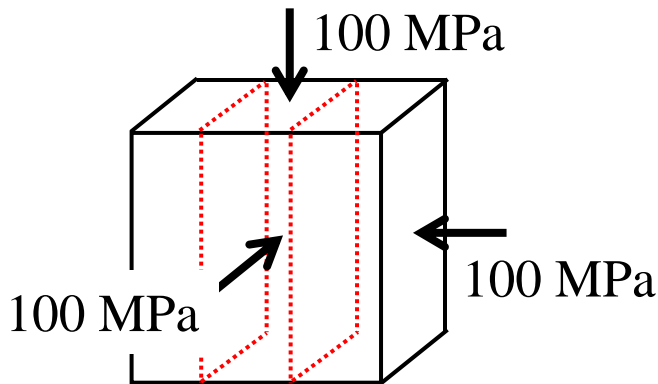
Property	Sandstone	<i>PFC</i> <sup>3D</sup> model
$E$ (GPa)	19.93	19.48
$\nu$	0.24	0.24
$\sigma_{ci}$ (MPa)	52	31
$\sigma_f$ (MPa)	112	109

# PFC3D Experiment

- Polyaxial cell mimics laboratory cell
- Failed when  $\sigma_1 > 120$  MPa
- Revised loading history:

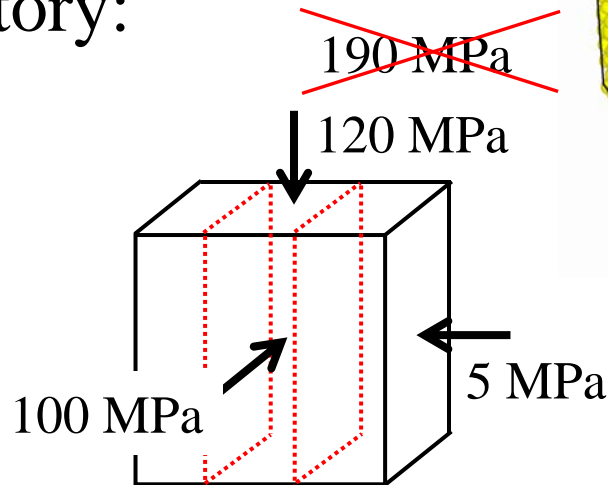


~20 grains  
21,899 pbonds



hydrostatic phase

168 + 7 bonds broke



fracturing phase

1056 bonds broke

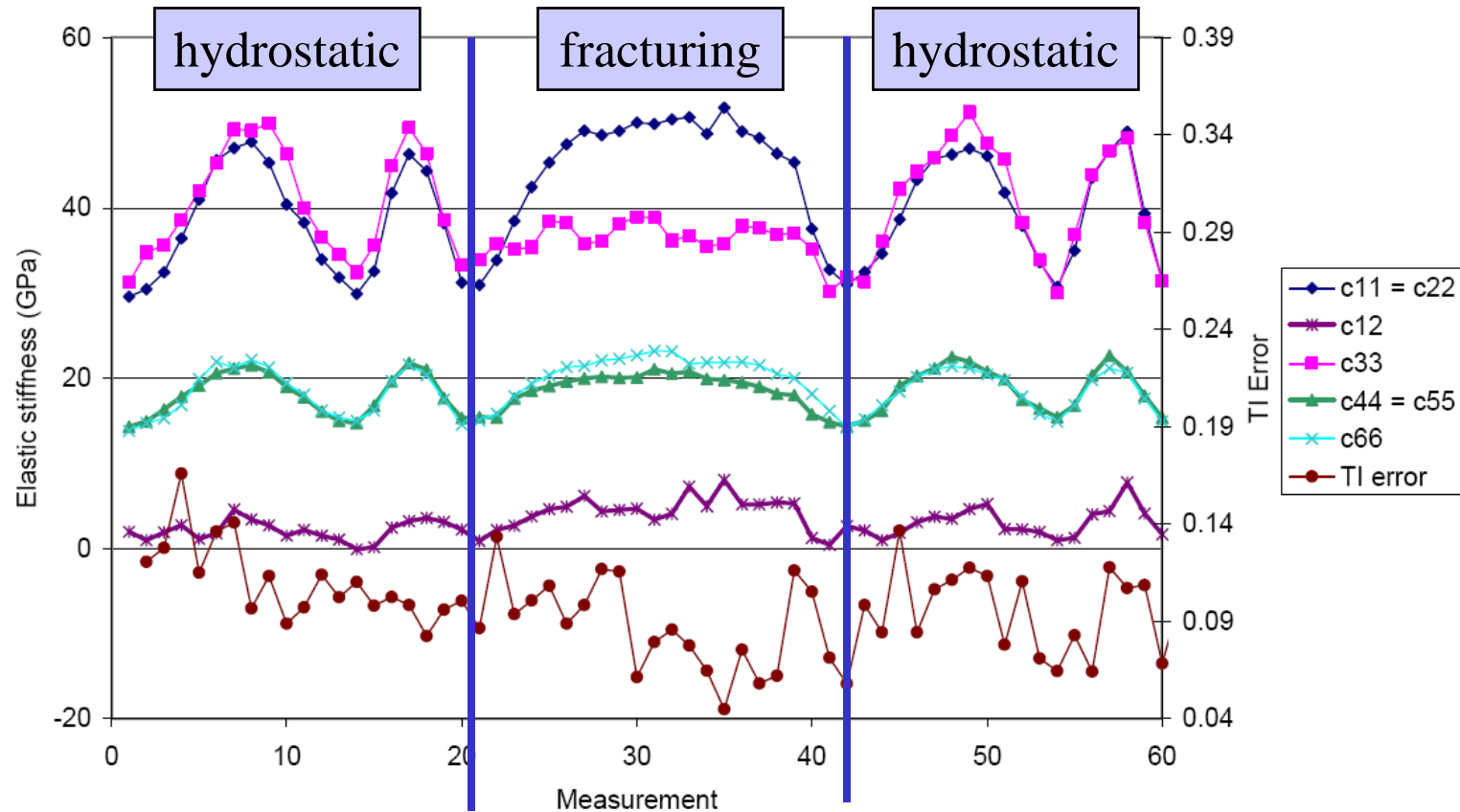
# Stiffness-Measurement Probes

- Stress probes (**static moduli**)
  - Load-unload cycles along each axis, measure stress/strain
  - Compute  $\{c_{11}, c_{12}, c_{13}, c_{33}\}$
- Wave-velocity probes (**dynamic moduli**)
  - Propagate waves along each axis, measure  $P_i$  and  $S_{ij}$
  - Compute  $\{c_{11}, c_{12}, c_{33}, c_{44}\}$
- Quantify goodness of fit of material response to transversely isotropic material model by **TI Error Index**:
  - Worst-case ratio of deviation of any meas. from its averaged value

# Laboratory Results

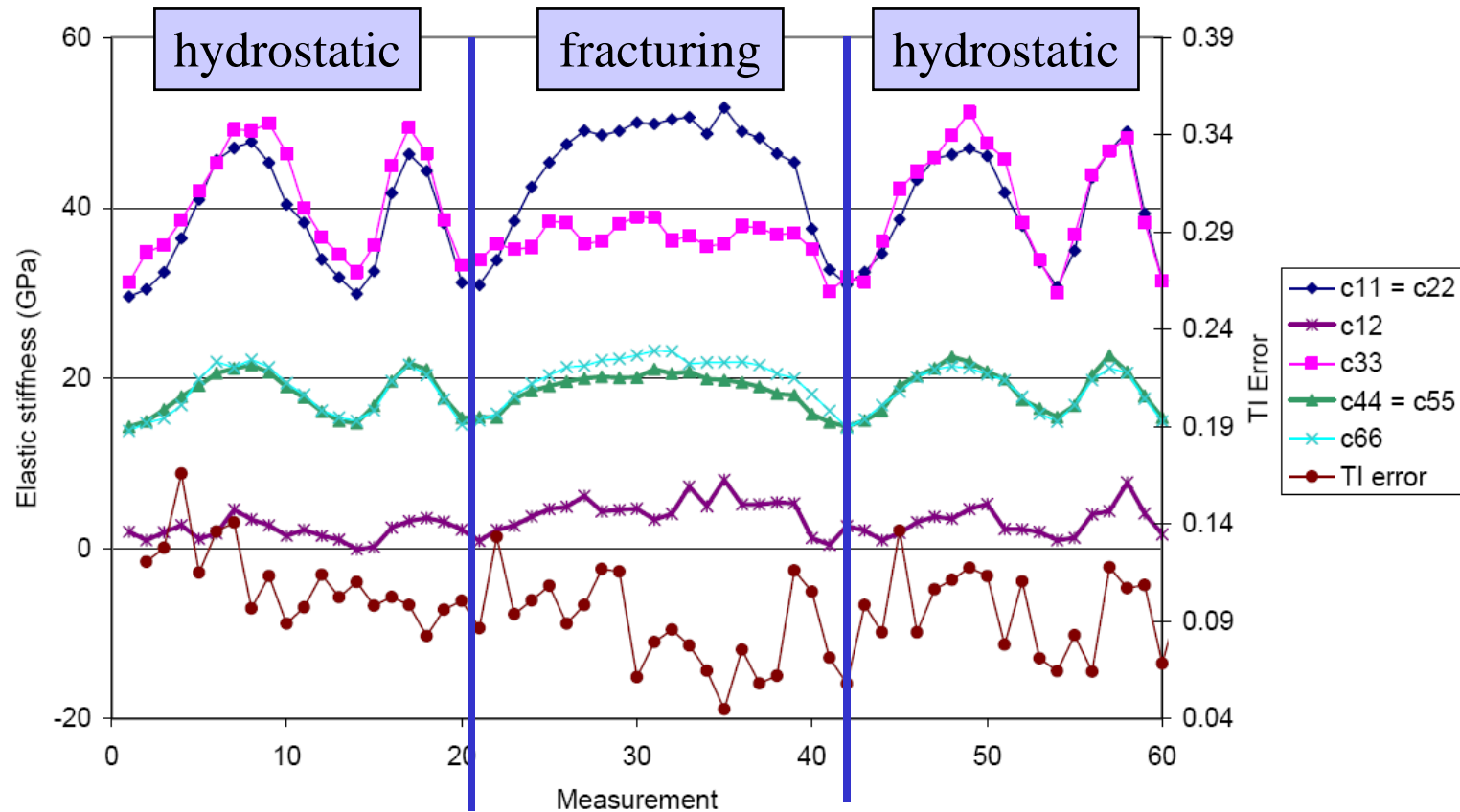
- **Stress probes** were unsuccessful
  - Time-dependency to sandstone response
    - stresses held constant for 30 sec., but straining continued
  - Insufficient strain-measurement resolution
    - Off-diagonal compliances not measured accurately
- **Wave-velocity probes** were successful...

# Laboratory Results



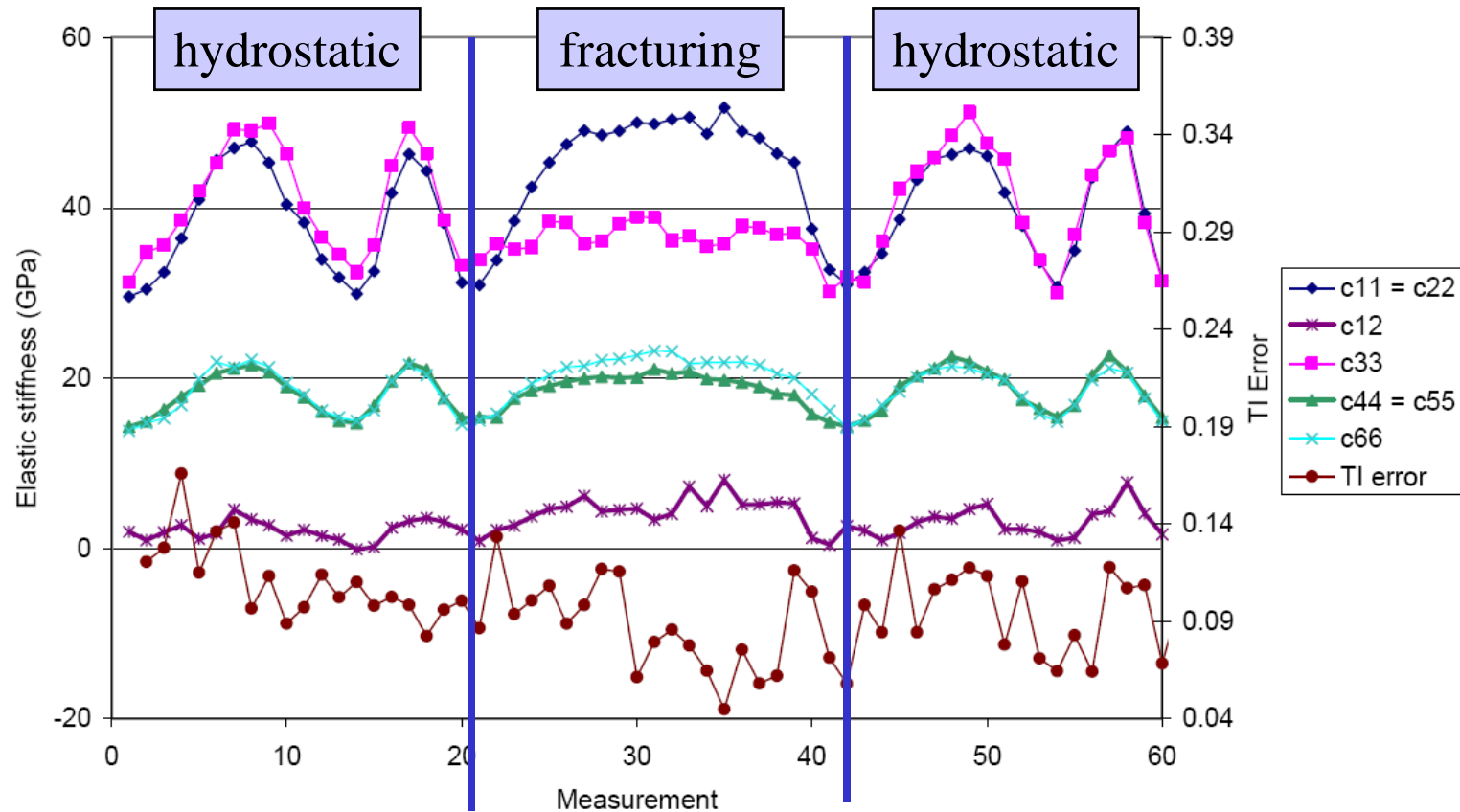
TI Error Index generally less than 15%  
[ Transversely isotropic material model is reasonable fit ]

# Laboratory Results



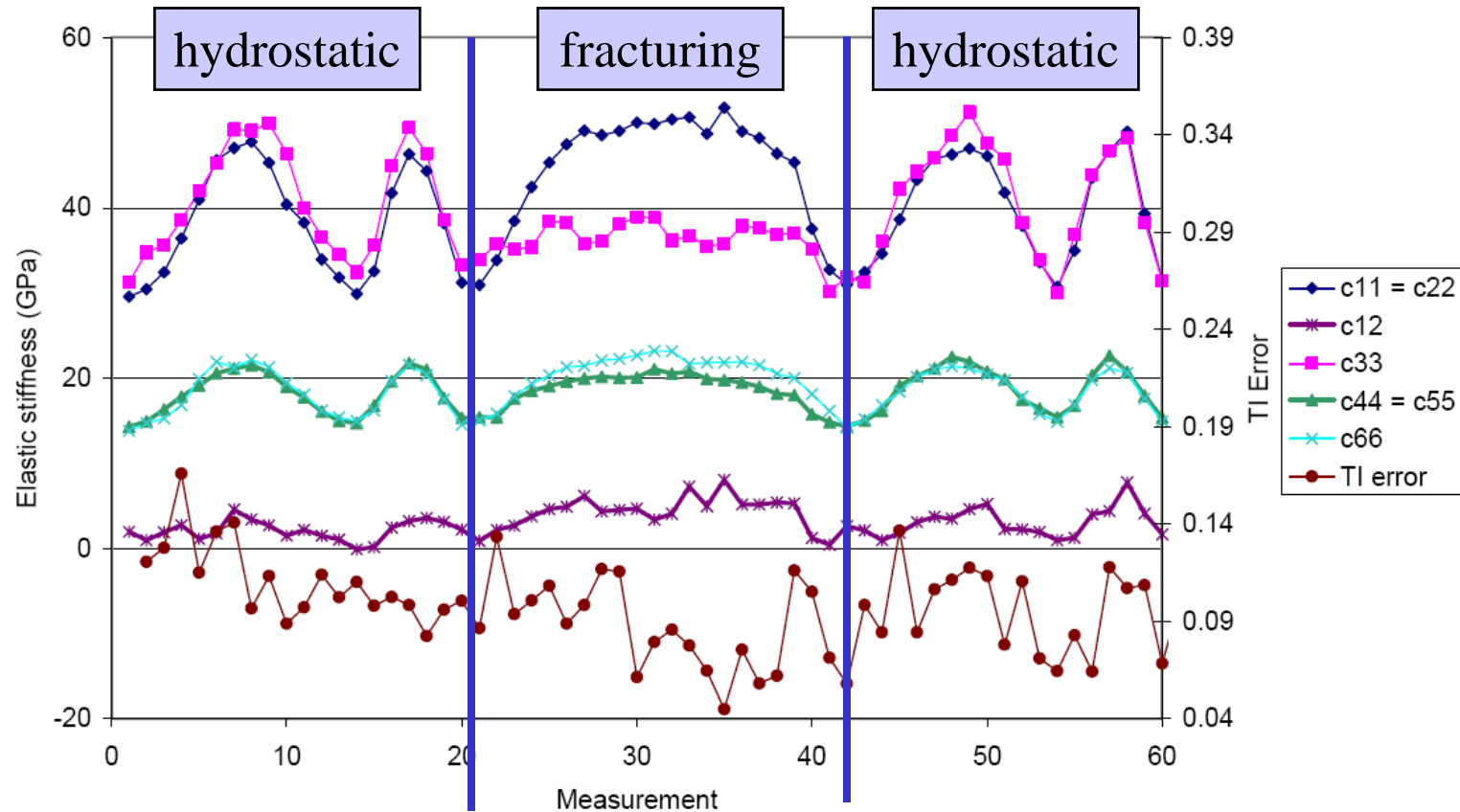
Large stiffness increase ( $c_{11}$  &  $c_{33}$ ) during hydrostatic loading  
[ pre-existing cracks or pores closing under stress ]

# Laboratory Results



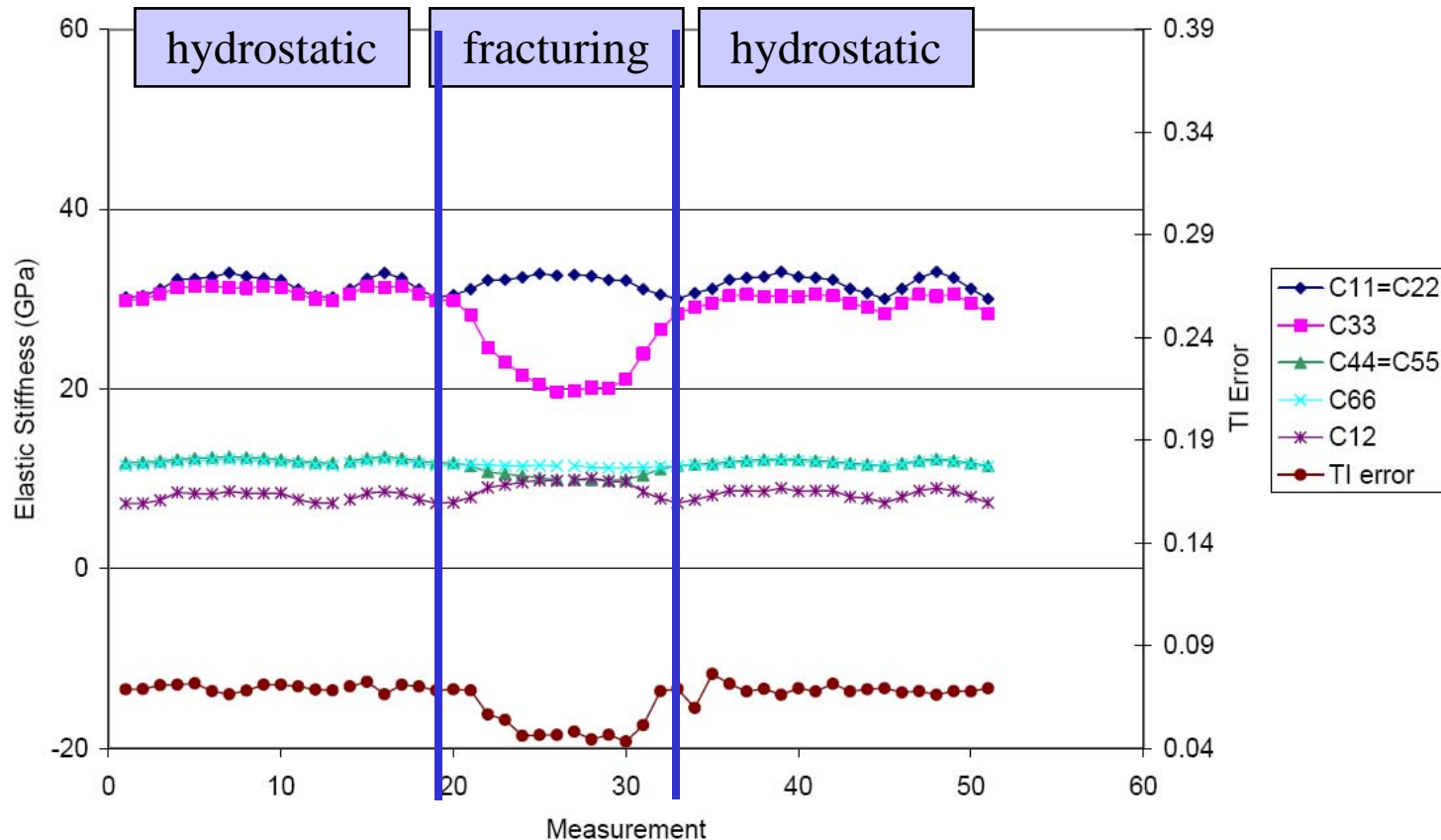
Damage induced during fracturing phase (measured AE), but only minor permanent stiffness ( $c_{33}$ ) reduction

# Laboratory Results



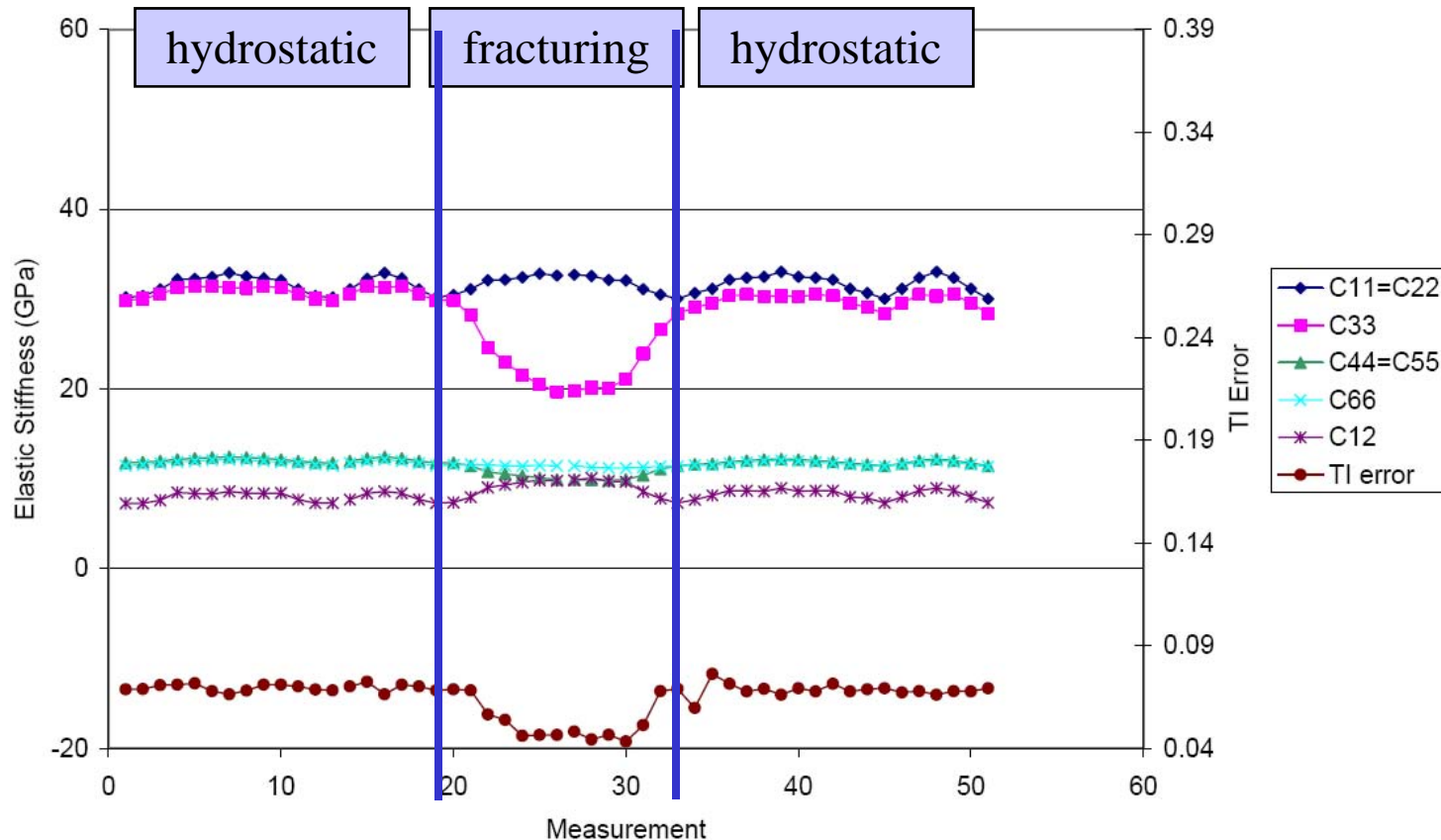
During fracturing phase, slight increase in stiffness ( $c_{33}$ )  
[ closing of pores has larger effect than opening of new cracks ]

# PFC3D Results (velocity probe)



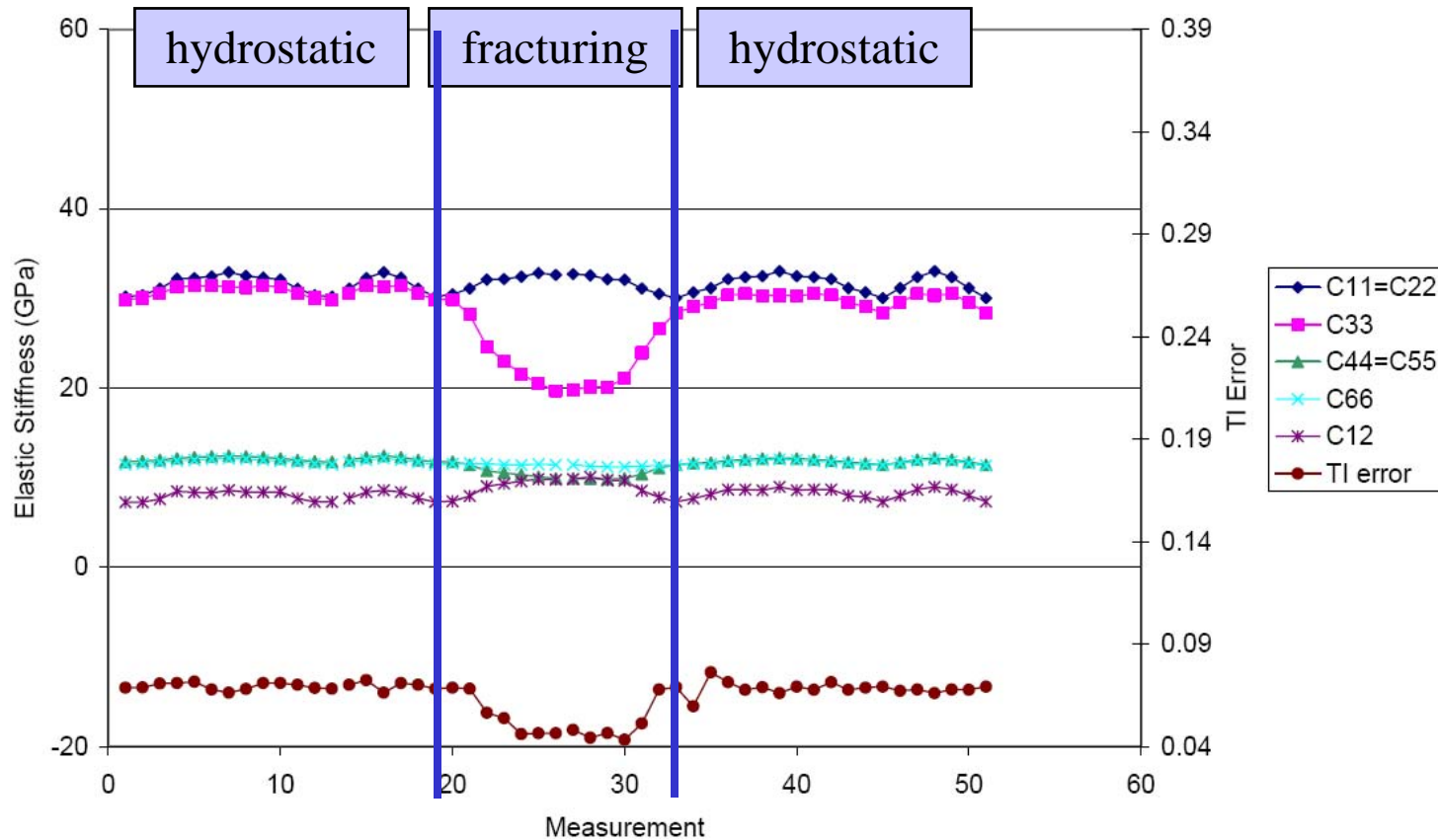
TI Error Index generally less than 8%  
[ Transversely isotropic material model is good fit ]

# PFC3D Results (velocity probe)



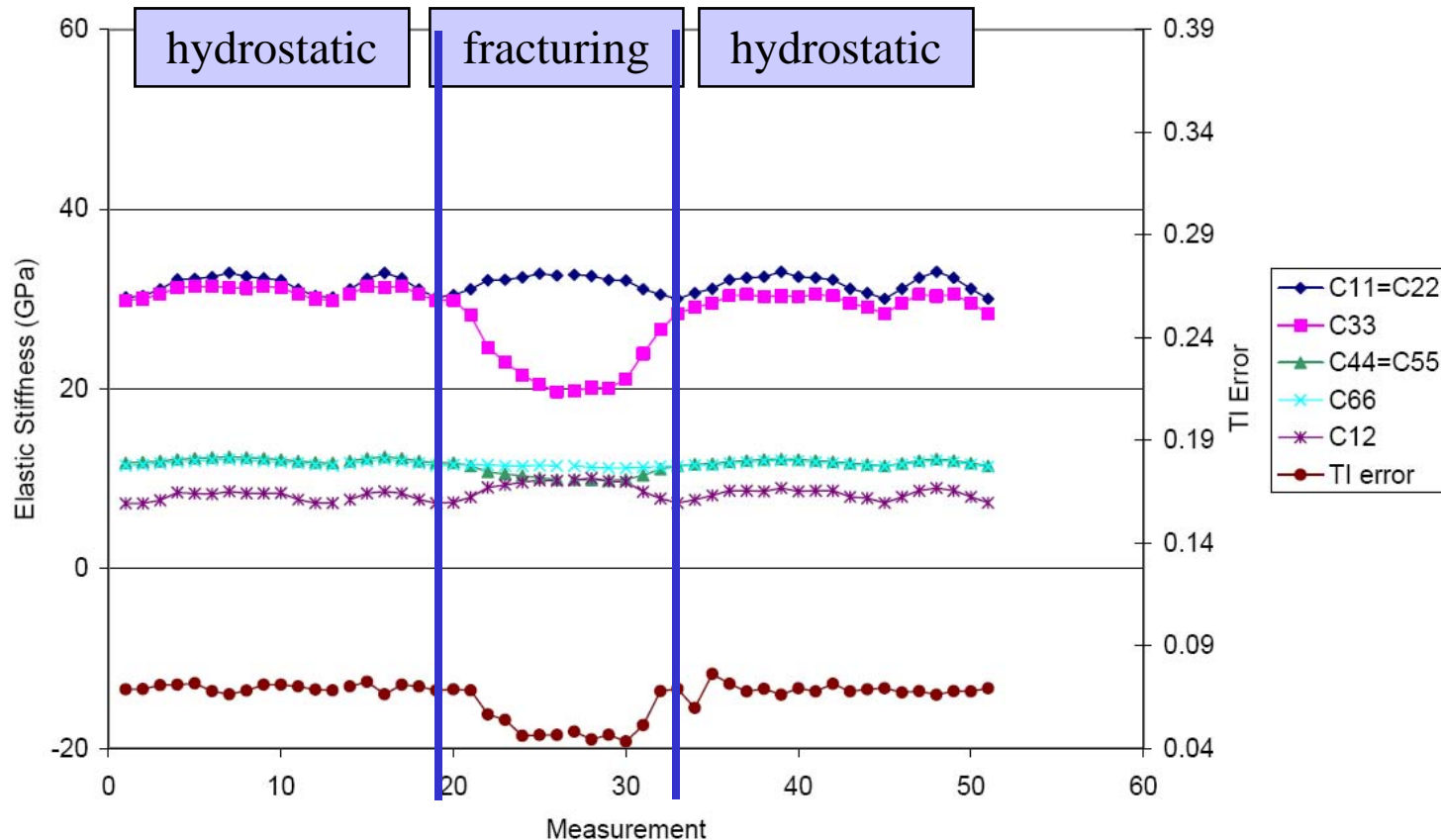
Small stiffness increase (c11 & c33) during hydrostatic loading  
[ absence of pre-existing cracks or pores,  
microstructure differs from the sandstone! ]

# PFC3D Results (velocity probe)



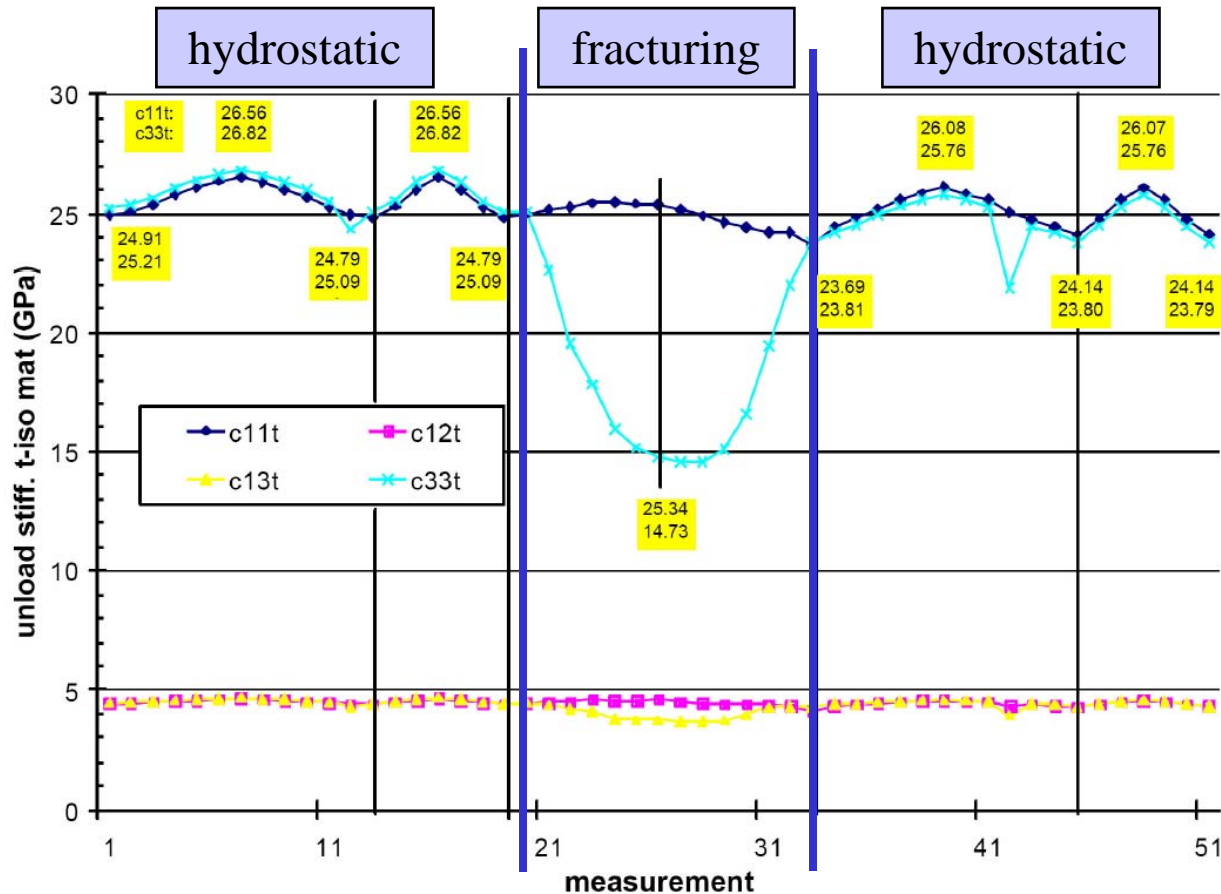
Damage induced during fracturing phase (1056 bonds broke), but only minor permanent stiffness (c33) reduction

# PFC3D Results (velocity probe)



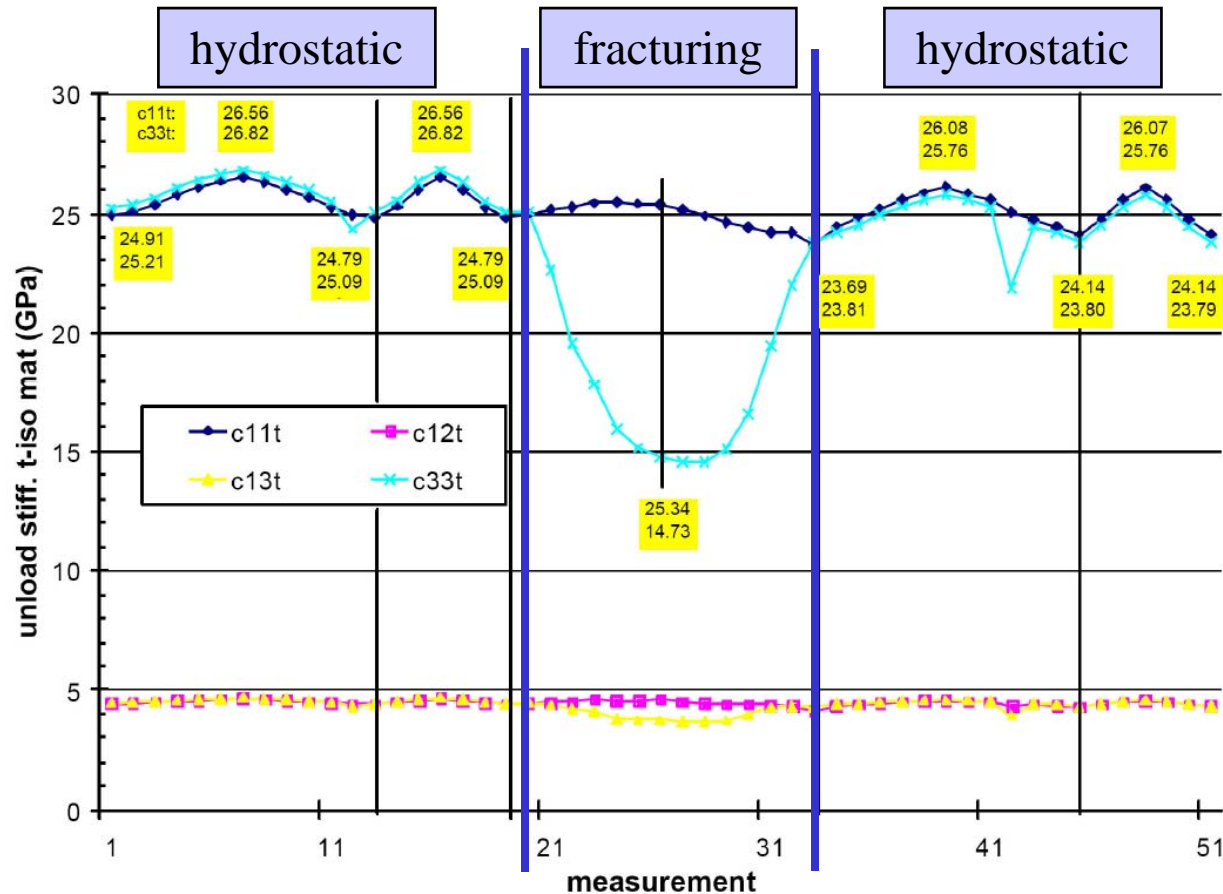
During fracturing phase, large decrease in stiffness (c33)  
[ from opening of cracks or deviatoric stress?,  
microstructure differs from the sandstone! ]

# PFC3D Results (stress probe)



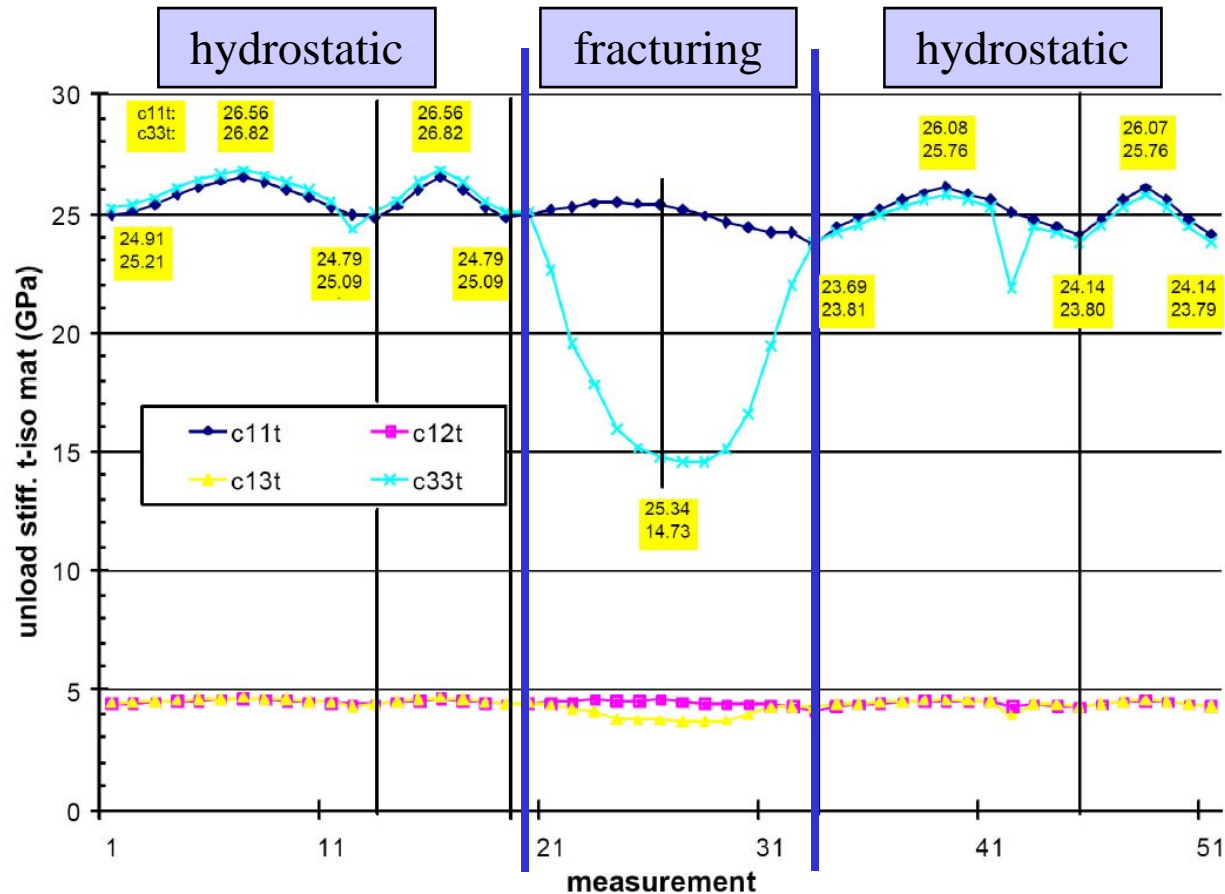
TI Error Index (not shown) generally less than 10%  
 [ Transversely isotropic material model is good fit ]

# PFC3D Results (stress probe)



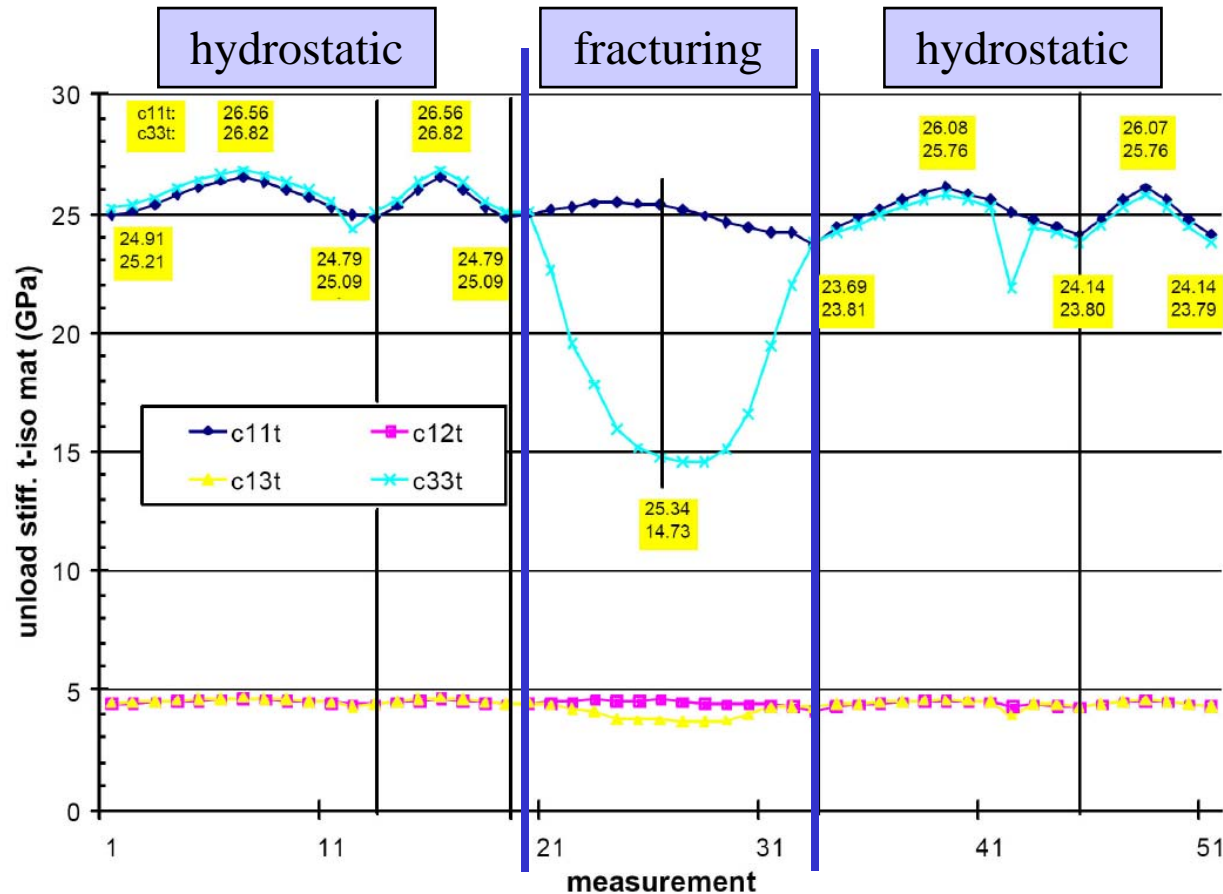
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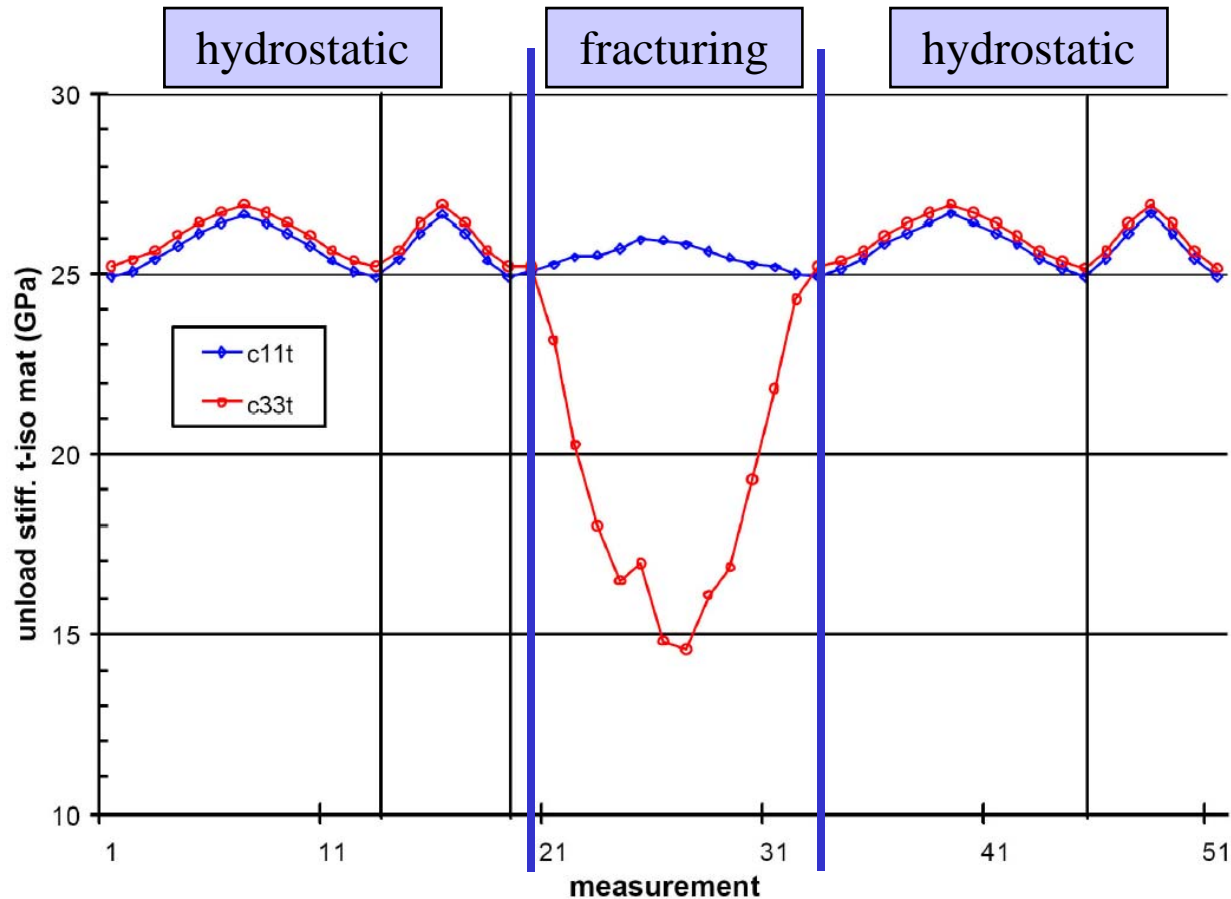
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# PFC3D Results (stress probe)



During fracturing phase, large decrease in stiffness (c33)  
[ from opening of cracks or deviatoric stress? ] →

# PFC3D Results (stress probe)



PFC3D material not allowed to crack (bonds cannot break).

Large anisotropy during fracturing phase is produced by deviatoric stress!

# Conclusions

- Static & dynamic results (PFC3D models) are similar, but dynamic stiffnesses are ~20% larger
  - Common observation in rock testing
  - Seismic pulses do not induce sliding on cracks
  - May be occurring in the model (did not confirm)
- Dynamic results (laboratory & PFC3D) show similar trends, but stiffness increase during hydrostatic loading much greater in lab test
  - Attributed to closure of pre-existing cracks & pores
  - A more realistic microstructure is required to reproduce this effect (perhaps, use smooth-joint logic to add pre-existing cracks)

# Conclusions

- Bonded-particle model with spherical grains joined by parallel bonds
  - Deviatoric loading induced temporary stiffness (c33) reduction of 41%
  - Bond breaks induced permanent stiffness (c33) reduction of 5%
  - Majority of stiffness change is produced by stress application and the induced change in number of contacts, not by creation of new cracks