Relating PFC parameters to rock properties for application to reservoir scale geomechanics

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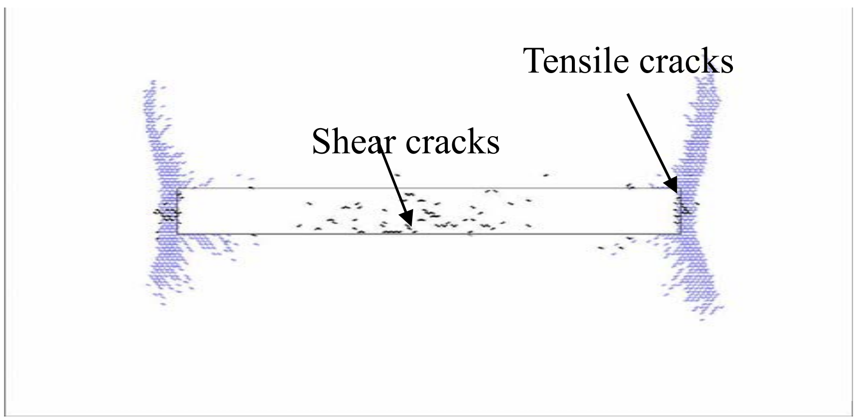
Modeling reservoir geomechanics using discrete element method (a preliminary study).

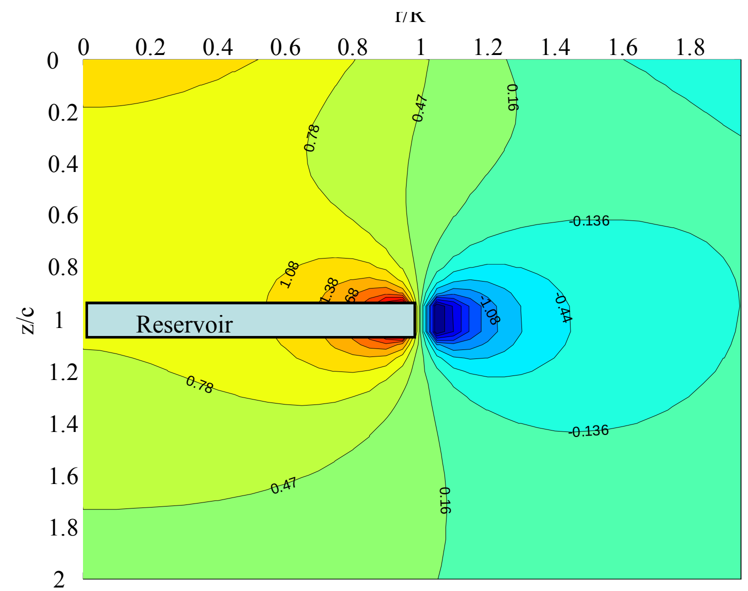
PFC2D model



Alassi et. al. 2006,

Pure & applied geophysics.

Stress concentration at the reservoir tip, analytical solution



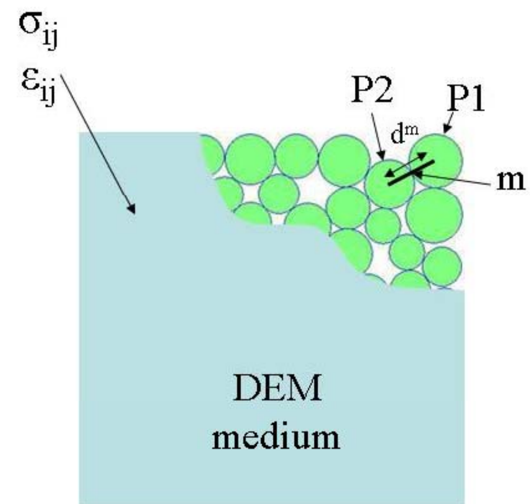
Motivation

- To use DEM (PFC) in modeling reservoir geomechanics and seismic wave propagation which is important for a full reservoir monitoring study. The advantage of DEM over other numerical models is its ability of modeling fracture propagation and fault reactivation in a dynamic way.

- To do this, **we need to relate PFC parameters directly to rock properties**, which are usually given for geomechanical and seismic data.

Derivation of a general equation (Walton, 1987)

Assuming that a granular medium is loaded from zero initial condition by stress σ_{ij} & strain ε_{ij}.



The stress in the granular medium can be given as

$$\sigma_{ij} = \frac{1}{2V}\sum_{m=1}^{Nc}(d_m I_i^m F_j^m + d_m I_j^m F_i^m)$$

We also get

$$\sigma_{ij} = \frac{1}{V}\sum_{m=1}^{N_C}\left(\frac{1}{2}(k_s^m \varepsilon_{jl} I_l^m I_i^m d_m^2 + k_s^m \varepsilon_{il} I_l^m I_j^m d_m^2) + (k_n^m - k_s^m)\varepsilon_{kl} I_i^m I_j^m I_k^m I_l^m d_m^2\right)$$

Remember also the relation for the elastic constitutive matrix C_{ijkl}

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$$

End up with

$$C_{ijkl} = \frac{1}{V}\sum_{m=1}^{N_C}\left(\frac{k_s^m d_m^2}{4}(I_j^m I_k^m \delta_{il} + I_i^m I_k^m \delta_{jl} + I_j^m I_l^m \delta_{ik} + I_i^m I_l^m \delta_{jk}) + (k_n^m - k_s^m) d_m^2 I_i^m I_j^m I_k^m I_l^m\right)$$

N_c: Number of contacts

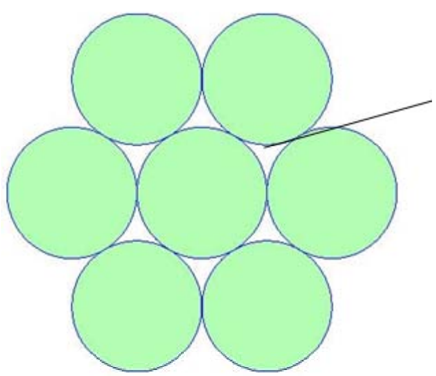
V: Medium volume

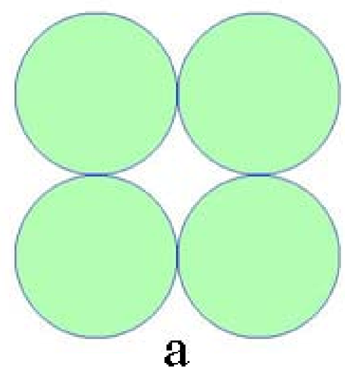
Dense Packing Vs. Loose random packing

1. Dense packing has high coordination number z (usually ≥ 4).

It follows that the ordered packing is a best example, like

- Hexagonal Packing
- Square Packing





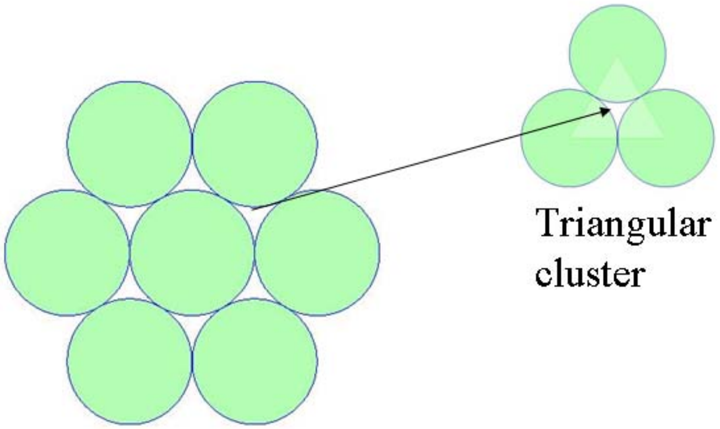
Hexagonal Packing

By using the general relation and setting N_c =3, V= area of the triangle we end up with

$$\lambda + 2\mu = C_{1111} = C_{2222} = \frac{\sqrt{3}}{4}(3k_n + k_s)$$

$$\mu = C_{1212} = C_{2121} = \frac{\sqrt{3}}{4}(k_n + k_s)$$

$$\lambda = C_{1122} = C_{2211} = \frac{\sqrt{3}}{4}(k_n - k_s)$$



Where λ, μ are the Lame's constants. The packing shows isotropic behavior, which was checked by looking to the other C_{ijkl} coefficients.

Hexagonal Packing, Dynamic Test

The packing porosity Φ and the packing density ρ can be given as

$$\Phi = 1 - \frac{\pi}{2\sqrt{3}} \qquad \rho = (1 - \Phi)\rho_s$$

Where ρ_s is the particle density

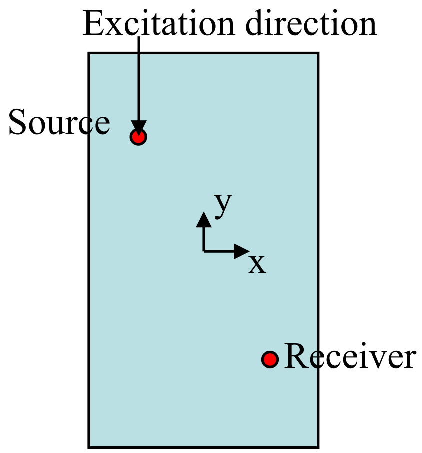
It follows that the P-wave and S-wave velocities can be given as

$$V_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{3}{2\pi\rho_s}(3k_n + k_s)}$$

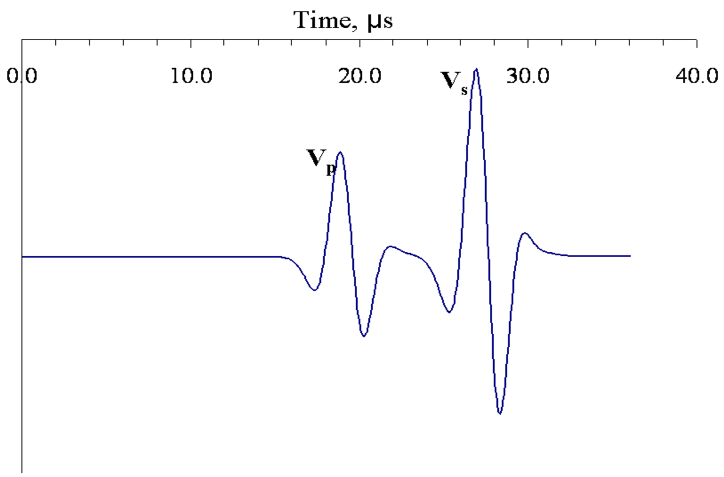
$$V_s = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{3}{2\pi\rho_s}(k_n + k_s)}$$

Dynamic Test, Homogeneous case

A vertical Ricker wavelet source signal is applied



Results in

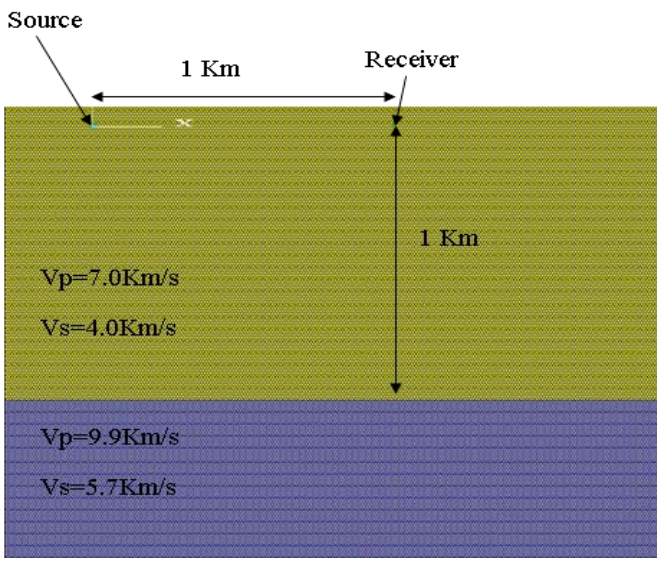


Property	k_n N/m	k_s N/m	ρ_s kg/m^3	V_p m/s	V_s m/s	V_p m/s	V_s m/s
				Analytical		Numerical	
Value	8.8e10	4.4e10	2630.0	7477	4895	7418	4887

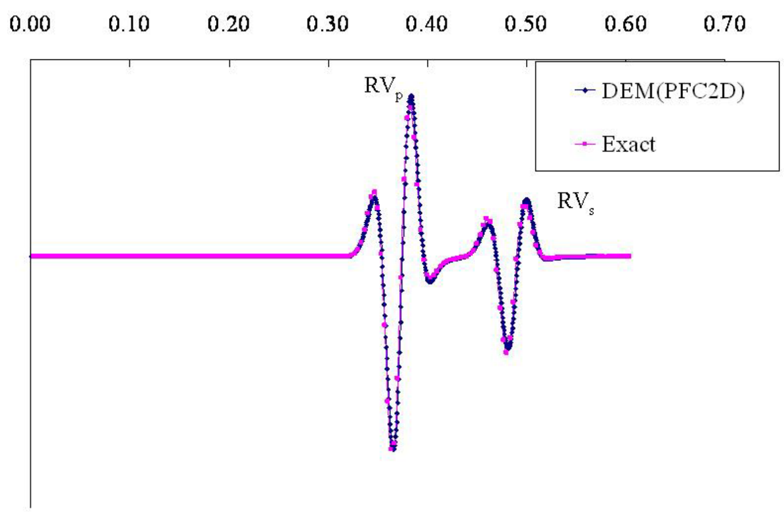
Dynamic Test, Heterogeneous case

A compressional Ricker wavelet source signal is applied in a Two-layers DEM Medium.

PFC2D Heterogeneous model



Reflected P-wave RV_p and reflected S-wave RV_s as obtained from PFC2D model and the exact Cagniard-De Hoop solution (De Hoop, 1960).



Hexagonal Packing, Static Test
Biaxial Test

Young's modulus E and Poisson's ratio ν for plane-stress condition can be given as

$$E = 2\sqrt{3}k_n \frac{k_n + k_s}{3k_n + k_s} \qquad \nu = \frac{k_n - k_s}{3k_n + k_s}$$

Biaxial test:

$$\nu = -\varepsilon_x/\varepsilon_y \qquad E = \sigma_y/\varepsilon_y$$

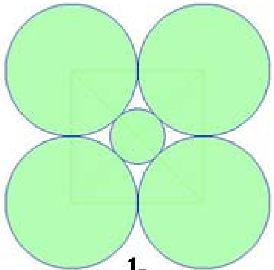
where ε_x is the lateral strain ε_x: vertical strain and σ_y: vertikal stress

The result as obtained from the PFC2D test and the analytical solution are

Property	k_n N/m	k_s N/m	E GPa	ν --	E GPa	ν --
			Analytical		Numerical	
Value	8.8e10	4.4e10	130.6	0.1428	130.3	0.1426

Square packing

A particle is added to the packing in oreder to get isotropic behaviour.



b

By using the general relation and setting N_c =8, V= area of the square we end up with

$$\lambda + 2\mu = C_{1111} = C_{2222} = \frac{1}{2}(3k_n + k_s)$$

$$\lambda = C_{1122} = C_{2211} = \frac{1}{2}(k_n - k_s)$$

$$\mu = C_{1212} = C_{2121} = \frac{1}{2}(k_n + k_s)$$

Square packing

- Dynamic test

$$\Phi = 1 - \pi(1 - \frac{1}{\sqrt{2}}) \qquad V_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{1}{\pi(2-\sqrt{2})\rho_s}(3k_n + k_s)} \qquad V_s = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{1}{2(2-\sqrt{2})\rho_s}(k_n + k_s)}$$

Property	k_n N/m	k_s N/m	ρ_s kg/m^3	V_p m/s	V_s m/s	V_p m/s	V_s m/s
				Analytical		Numerical	
Value	8.8e10	4.4e10	2630.0	7977	5222	7871	5148

- Static test

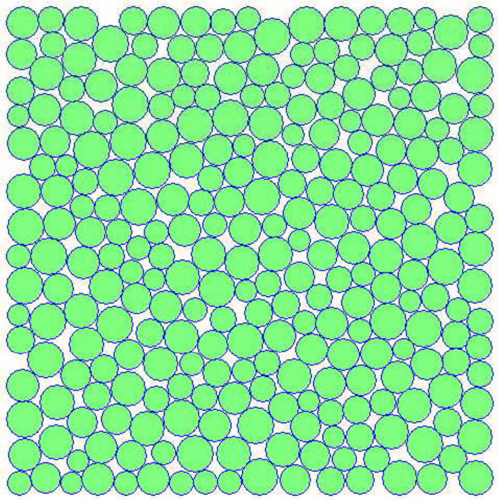
$$E = 4k_n \frac{k_n + k_s}{3k_n + k_s} \qquad \nu = \frac{k_n - k_s}{3k_n + k_s}$$

Property	k_n N/m	k_s N/m	E GPa	ν --	E GPa	ν --
			Analytical		Numerical	
Value	8.8e10	4.4e10	150.8	0.1428	150.3	0.1425

Dense Packing Vs. Loose random packing

2. Random loose packing

By using the general equation we end up with:



$$\lambda + 2\mu = C_{1111} = C_{2222} = \frac{(1-\Phi)z}{4\pi}(3k_n + k_s)$$

$$\lambda = C_{1122} = C_{2211} = \frac{(1-\Phi)z}{4\pi}(k_n - k_s)$$

$$\mu = C_{1212} = C_{2121} = \frac{(1-\Phi)z}{4\pi}(k_n + k_s)$$

Where the porosity Φ and the coordination number z is defined as

$$\Phi = 1 - \frac{N_p \pi d_{av}^2}{4V}$$

$$z = \frac{2N_c}{N_p},$$ where N_p is the number of particles.

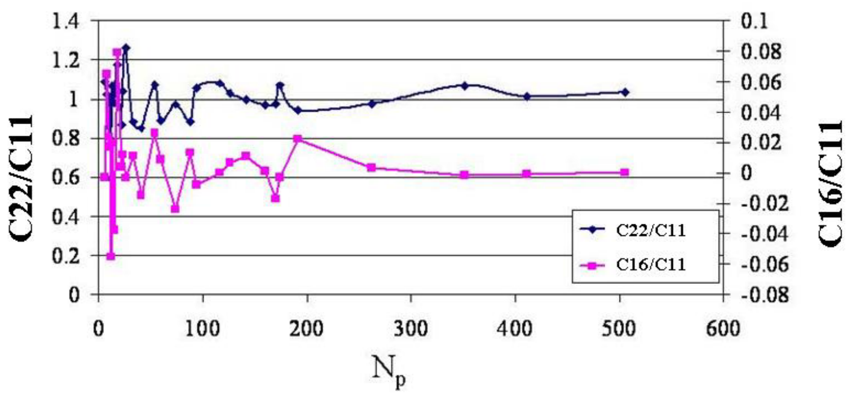
Random loose packing
Dynamic Test

P-wave and S-wave velocities can be given as

$$V_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{z}{4\pi\rho_s}(3k_n + k_s)} \qquad V_s = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{z}{4\pi\rho_s}(k_n + k_s)}$$

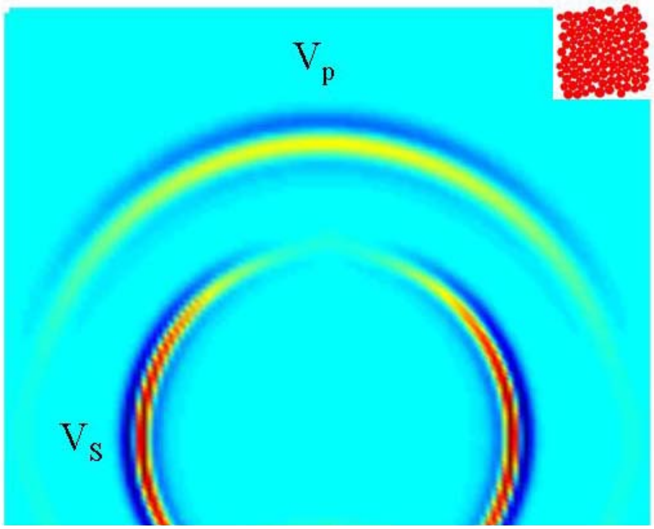
From above equations we get $V_p/V_s \leq \sqrt{3}$

The model is built out of Pbricks, where the number of particles per Pbrick should be sufficient to give isotropic behavior. The following figure may be used as a guidance



Random loose packing, Dynamic Test

Wave front propagation through the model.

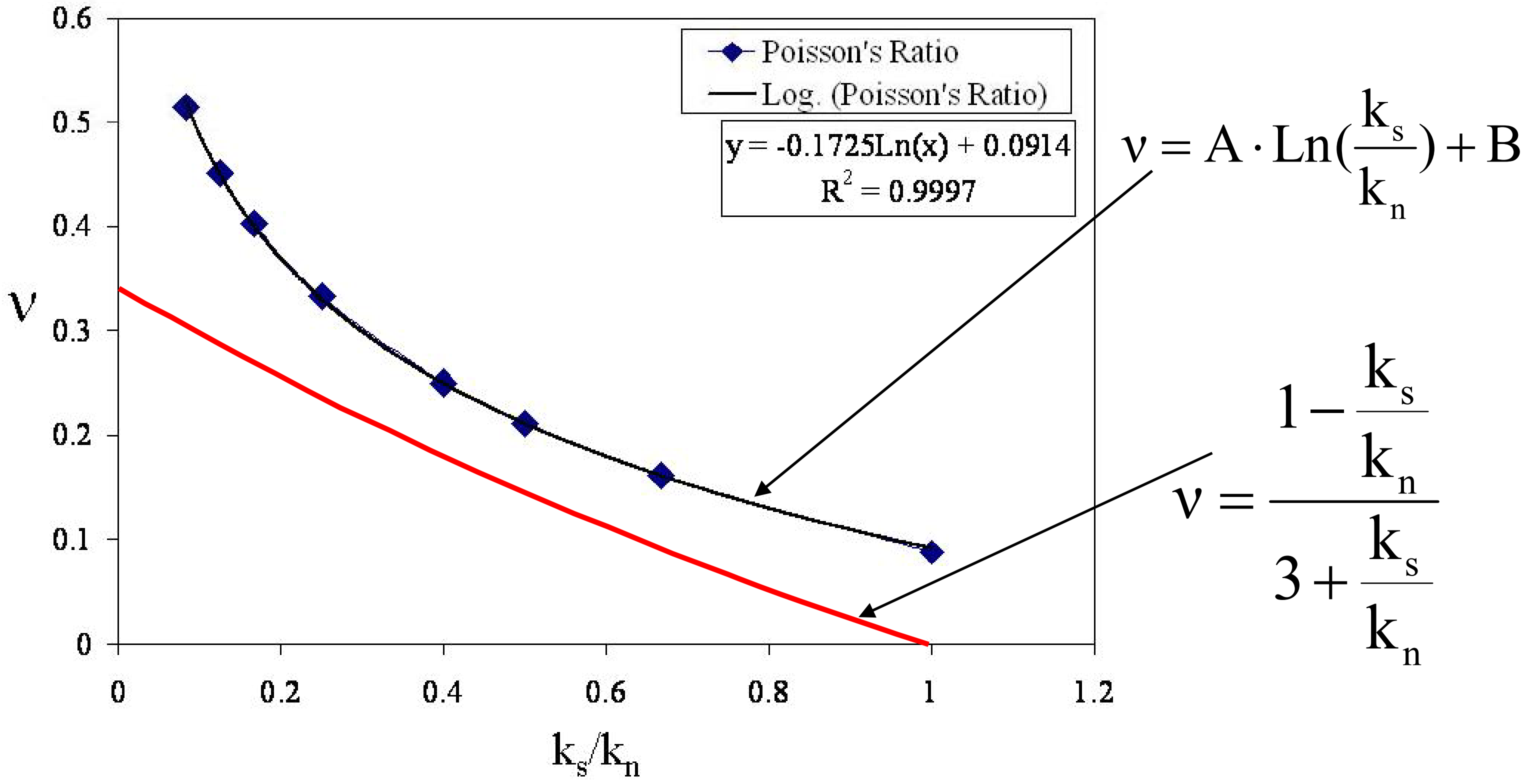


Modeling Result

Property	k_n N/m	k_s N/m	z --	ρ_s kg/m^3	V_p m/s	V_s m/s	V_p m/s	V_s m/s
					Analytical		Numerical	
Value	30e9	15e9	3.4	2630	3286	2151	3111	2040

Random loose packing, Static Test

By using biaxial test, it is shown that the static behavior for the random loose packing differs from the dense packing.



Effect of particles rotation, Comparison to Cosserat continuum

The correct expression for the stress tensor which includes the particle rotation effect should be written as

$$\sigma_{ij} = \frac{1}{V}\sum_{m=1}^{N_C}\left(\frac{k_s^m d_m}{2}(\Delta U_j^m I_i^m + \Delta U_i^m I_j^m) - \frac{k_s^m d_m^2}{2}(I_i^m I_k^m \theta^m + I_j^m I_k^m \theta^m)\right.$$
$$\left. + (k_n^m - k_s^m) d_m \Delta U_k^m I_k^m I_i^m I_j^m \right)$$

The wave equation is given as

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{\partial^2 u_i}{\partial t^2}$$

where

$$\Delta U_i^m \approx \frac{\partial u_i}{\partial x_j}\Delta X_j^m = \frac{\partial u_i}{\partial x_j} I_j^m d^m$$

So we get

$$(3k_n + k_s)\frac{\partial^2 u_1}{\partial x_1^2} + 2k_n \frac{\partial^2 u_2}{\partial x_2 \partial x_1} + (k_n + k_s)\frac{\partial u_1^2}{\partial x_2^2} - 4k_s \frac{\partial \theta_{21}}{\partial x_2} = \frac{4\pi\rho_s}{z}\frac{\partial u_1^2}{\partial t^2}$$

The wave equation for the Cosserat continuum can be given as (see Mindlin, 1965)

$$(\lambda + 2\mu)\frac{\partial^2 u_1}{\partial x_1^2} + (\lambda + \mu - \beta)\frac{\partial^2 u_2}{\partial x_2 \partial x_1} + (\mu + \beta)\frac{\partial u_1^2}{\partial x_2^2} - 2\beta \frac{\partial \theta_{21}}{\partial x_2} = \rho \frac{\partial u_1^2}{\partial t^2}$$

Effect of particles rotation, Comparison to Cosserat continuum

By comparing the previous 2 equations

$$\beta = \frac{(1-\Phi)z}{4\pi} 2k_s$$

$$V_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} = \sqrt{\frac{z}{4\pi\rho_s}(3k_n + k_s)}$$

$$V_s = \sqrt{\frac{\mu + \beta}{\rho}} = \sqrt{\frac{z}{4\pi\rho_s}(k_n + k_s)}$$

If the particle rotation is constrained ($\theta = 0$),we get (c stands for constrained)

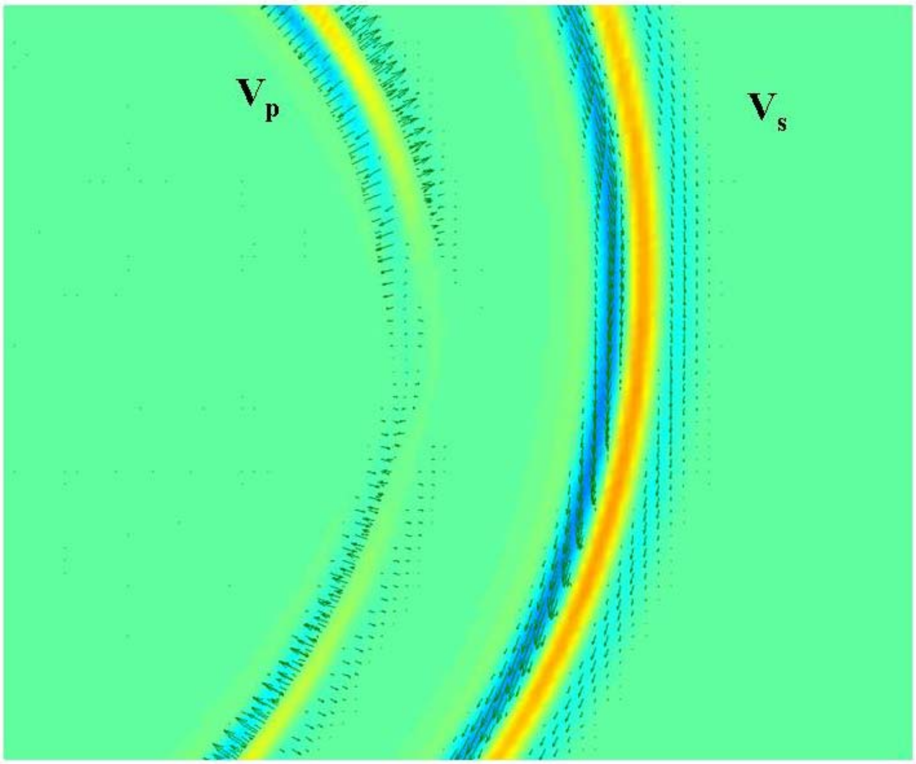
$$\mu^c = \frac{(1-\Phi)z}{4\pi}(k_n + k_s) \qquad \lambda^c = \frac{(1-\Phi)z}{4\pi}(k_n - k_s)$$

So we end up with

$$V_p = \sqrt{\frac{\lambda^c + 2\mu^c}{\rho}} = \sqrt{\frac{z}{4\pi\rho_s}(3k_n + k_s)}$$

$$V_s = \sqrt{\frac{\mu^c + \beta}{\rho}} = \sqrt{\frac{z}{4\pi\rho_s}(k_n + 3k_s)}$$

Wave propagation front moving in a model where a particle rotation is constrained, notice how the S-wave travels faster than the P-wave which is unrealistic for isotropic elastic material.

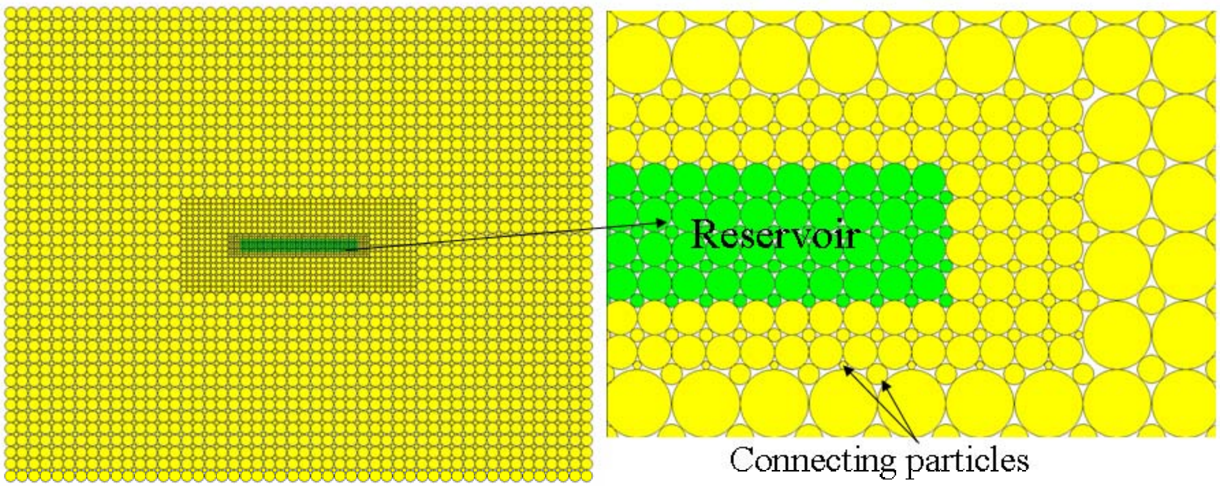


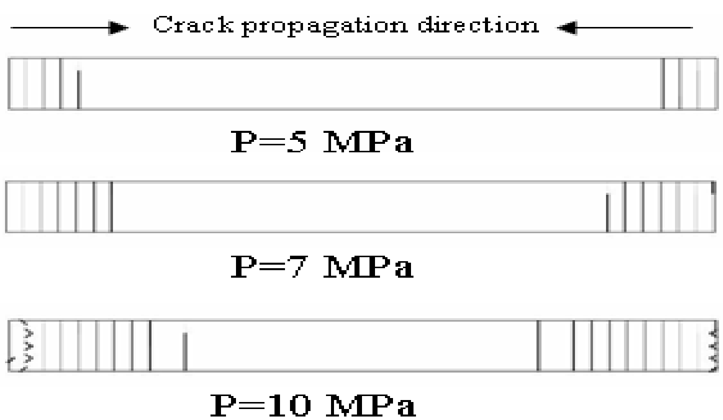
$k_s > 3k_n$

Conclusions

- For elastic case, analytical and empirical relations for rocks can be obtained where good agreements between these relations and PFC numerical test are achieved.
- The static and the dynamic behavior for dense packing are the same.
- The maximum P-wave S-wave ratio is limited by ($V_p/V_s<\sqrt{3}$).
- The static behavior for the random loose packing differs from the dynamic one where the limitation on ν ($< 1/3$) is no longer valid.
- The particle rotation affects the dynamic behavior where if the particle rotation is constrains the S-wave can propagate faster than the P-wave.

Crack propagation during pressure increase inside a rectangular reservoir



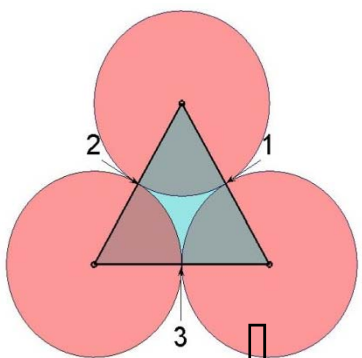


$\sigma'_h = 2$ MPa

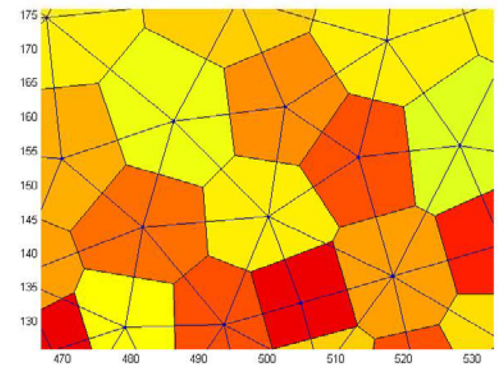
$\sigma'_v = 10$ MPa

Theory of modified DEM

Original DEM



• Finally

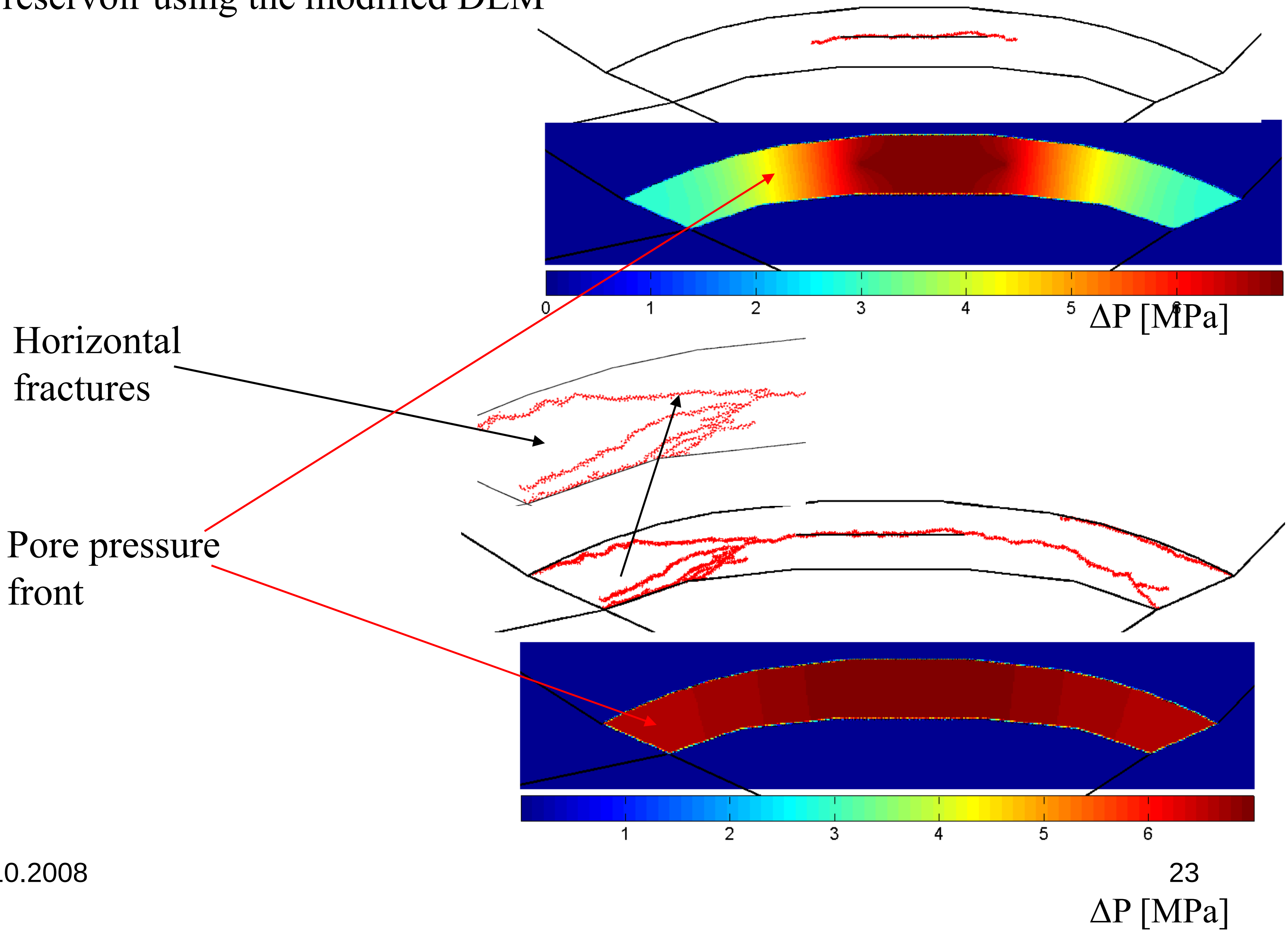


Using this equitation

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} K1 & 0 & 0 \\ 0 & K2 & 0 \\ 0 & 0 & K3 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} K1 & a1 & a2 \\ a3 & K2 & a4 \\ a5 & a6 & K3 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

Fracture development during fluid injection inside a hydrocarbon reservoir using the modified DEM



Horizontal fracture scenario